

# **Computing Binary Star Observables**

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## 1. Introduction

The model has been described and quantified in papers by Wilson & Devinney (1971), Wilson (1979, 1990, 1993), and Van Hamme & Wilson (2003, 2007), that include the model’s main theory, organization, and concepts, as well as much of the mathematics. The reader is referred to those papers for background. Since it would be impractical to cover the programming ideas within a reasonable length, only the 1993 paper goes significantly into programming, and there only in abbreviated form. This booklet also is not an explanation of programming ideas – its purpose is to tell how to use the model.

The overall program consists of a main FORTRAN program (**LC**) for generating light and radial velocity curves, spectral line profiles, and images, plus a differential corrections main program (**DC**) for parameter adjustment of light and velocity curves by the Least Squares criterion, and somewhat over two dozen subroutines used by both main programs.<sup>1</sup> The present program has grown over the years from much simpler beginnings. Growth has consisted of occasional improvements in regard to generality, speed, and elimination of bugs, perhaps about every six months, punctuated by five revisions. The revision of 1982 introduced eccentric orbits, asynchronous rotation, capacities to do several new kinds of constrained solutions, computation of velocity curves (with proximity and eclipse effects), simultaneous light and radial velocity solutions, and a simple star spot capability. Most features of the 1982 version are described in Wilson (1979). The revision of 1992 had options for detailed reflection and non-linear (“logarithmic” law) limb darkening, adjustment of spot parameters, an optional provision for spots to move with the rotating surface, capability for following light curve development over large numbers of orbits, and greater speed. At that time an accuracy improvement was expected for the near term, but insufficient resources have been available for completion of that project, which is still on the “back burner.” Instead, the third revision (1998) included semi-transparent circumstellar clouds, a simple spectral line profile capability for fast-rotating stars, inclusion of the Marquardt  $\lambda$  factor in differential corrections solutions, generation of sky coordinates for images, an option to work with either observed times or phases, additional solution parameters ( $T_0$ ,  $P_0$ ,  $dP/dt$ , and  $d\omega/dt$ ), conversion of the entire program to double precision, and some other improvements that are listed in §19, pp. 42-43. An accuracy improvement still is expected, but the effective date is hard to predict. The fourth revision (2003) was mainly a conversion from the previous effective wavelength characterization of bandpasses to one based on integrations over actual bands of standard photometric systems. The radiative functions depend on effective temperature,  $\log g$ , and chemical composition and are applied locally. A few changes also were made in the constraints applied to some overcontact binaries (mode 3). This fifth revision of 2007 includes light-time and velocity shifts due to third bodies (hereafter 3b’s), adjustment of corresponding 3b parameters (§7.5, p. 18), computation of  $x, y$  limb darkening coefficients as functions

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<sup>1</sup>In this monograph, program and subprogram names are in boldface type (e.g. **LC**), FORTRAN variable names are in sans serif style (e.g. XINCL), and FORTRAN statements and file names are in typewriter style (e.g. `open(unit=22,...)`).

of local  $T_{eff}$ ,  $\log g$ , and  $[M/H]$  for any of three laws (§7.4, pp. 18-18), option of magnitude or flux input to **DC** (see explanation of control integer **MAGLITE** in §9.3, p. 26), simplified re-dimensioning (provided by A. Prša), and several additions and improvements in output quantities and formats. Instructions for re-dimensioning are in the **LC** and **DC** main programs.

## 2. Running the Programs

To make things simple, the light and velocity curve program (**LC**) and the differential corrections program (**DC**) are supplied complete with sample input data sets, so that all one needs to do to get started is to run the programs with the input data. There should be few, if any, machine-dependent problems, as those have been eliminated via feedback from users. If such a problem is found, please communicate it to R.E.W. However, there may be a trivial problem with the arcsine and arccosine routines, which are **DASIN** and **DACOS** on some systems and **DARSIN** and **DARCOS** on others. That problem can easily be fixed. Then just change the numbers to run a particular binary star problem. Be sure to keep at least one copy of the supplied sample data in case the input formatting gets scrambled or shifted. Of course, one always can re-construct the correct format by comparing the program's **READ** and **FORMAT** statements with the input lines, but save the original data anyway in the interest of keeping things simple. It is not recommended to change the programs, but if you must, be sure to keep a copy of the original version for comparison. Tinkering can introduce a bug that may not show up in particular cases, but jumps out at you later on.

Compilation is quite direct as **LC** is supplied in one large file and so is **DC**, including all required subroutines. This arrangement differs from the 1998 and earlier versions where there was one large file that had to be re-arranged into **LC** and **DC** modules prior to compilation. The new way should be more convenient. Most subroutines are in both modules, with memory cost of the redundancy being of little significance.

The type of output produced by **LC** is determined by control integer **MPAGE** and can be light curves (**MPAGE**=1), radial velocity curves (**MPAGE**=2), spectral line profiles (**MPAGE**=3), relative star dimensions (**MPAGE**=4), or sky coordinates for producing images (**MPAGE**=5).

For **MPAGE**=1 (light curves), the first five output columns in the block after the header information contain time (col. 1), phase (col. 2), separate light for stars 1 and 2 (cols. 3, 4), and the combined system light from stars 1 and 2 plus third light (col. 5). **Star 1 is, by definition, the one at superior conjunction near phase zero when parameter PSHIFT is entered as zero.** It will be the one eclipsed near phase zero if there are eclipses. Light is in the program unit, which is explained in Wilson (1993). The program unit is, in a sense, an absolute unit of observable flux because it can be converted to standard physical units if absolute star luminosities and a definite observer distance are specified. Light (observable flux) and luminosities are discussed in §4, p. 9. Column 6 contains the light of column 5 re-scaled (normalized) to a specified input value (labeled **FACTOR**) at a specified input phase (labeled **PHN**). For example, one can require that

the normalized light be 1.2000 at phase 0.1500. This re-scaling provision is only for convenience in working with graphs, and has no relevance to differential corrections solutions. That is, the **DC** program works only with direct flux (column 5 of **LC** output) and does not even know about normalized light. Column 7 is the star separation, with the unit being the relative orbital semi-major axis ( $a = a_1 + a_2$ ). Column 8, called “set-level magnitude,” is system brightness expressed in magnitudes, pinned so as to pass through a definite point in a graph of magnitude vs. phase. The point is specified by input quantities **MZERO** and **PHN**. For example, if **MZERO** is +7.300, column 8 will read 7.300 at the phase of normalization (**PHN**), which is the same phase of normalization used for column 6. Column 9, called “direct magnitude,” is  $-2.5 \log_{10} \ell$ , which is the exact inverse of flux used by **DC** for its solutions, as generated in **DC** by  $\ell = 10^{-0.4m}$  in the case of magnitude input. Here  $\ell = \ell_1 + \ell_2 + \ell_3$  (column 5). Accordingly, **LC**’s direct magnitudes will give theoretical light curves that correspond to **DC** solutions and can be used to make graphs to compare theory with observation. Column 10 is the light-time shift ( $\Delta t$ , in days) caused by the third body.

For **MPAGE=2** (radial velocities), the first two output columns are again time and phase. Columns 3 and 4 are dimensionless radial velocities for the two stars (in circular relative orbit circumference,  $2\pi a$ , per orbit cycle). Columns 5 and 6 are the eclipse-proximity corrections corresponding to columns 3 and 4, respectively, and in the same dimensionless unit. Columns 7 and 8 are velocities in kilometers per second if velocity unit **VUNIT** was entered as 1.00, or, in general, in unit **VUNIT** km/s. Column 9 is the light-time shift and column 10 the radial velocity shift caused by the third body.

For **MPAGE=3** (spectral line profiles), the output is by blocks according to phase, with results for stars 1 and 2 given first and second, respectively, within each phase block. Column 1 is the equivalent velocity difference between the reference wavelength and the wavelength of the profile point, in unit **VUNIT**. Column 2 is the corresponding wavelength difference, in microns. The reference wavelength is that entered with the main binary star parameters (labeled “wv lth” in the output). Column 3 has the actual wavelengths of the profile points, in microns. Column 4 is the profile in terms of a flat continuum at flux 1.0000000. Column 5 is the profile in terms of a continuum that can be shifted vertically and can have a slope. Guard against generating excessive output with **MPAGE=3** (usually one does only one phase at a time). Input for **MPAGE=3** operation is described in §6, p. 11. It requires extra data lines beyond those for other **MPAGE** values. The program will crash without those lines.

For **MPAGE=4** (star figures), **LC** lists the pole, point, side, and back radii of each star vs. time and phase. This provision can be useful for eccentric orbit cases, as it shows the variation of figure with phase.

For **MPAGE=5** (images), **LC** produces only two output columns for each time/phase, beyond the usual header information. Those columns are  $y_{\text{sky}}$  and  $z_{\text{sky}}$  rectangular plane of sky coordinates of the projected surface elements of the two stars. A picture of the binary at a given phase can be made by sending just those two columns to a plot program such as **GNUPLOT**, **MONGO**, etc. Any spots or parts of spots that may be in view will show in the pictures, although there is no



distinction in the pictures between bright and dark spots. The images show only spot location, not spot surface brightness (there is no gray scale). The origin (0,0) of the image coordinates is at the binary system center of mass.

Adjustment of parameters while fitting light and velocity curves normally involves both subjective (**LC**) and objective (**DC**) iteration. It is assumed that the user has reasonable background knowledge of binary stars and can show good judgment in deciding which parameters to fix from theory, which to fix from other kinds of observations, and which to adjust. Solution constraints, if there are to be any, must also be decided upon. In most situations, some of the decisions are obvious while others are debatable, and seldom will two persons make the same set of choices, even if given exactly the same circumstances. Therefore, this section goes only into the “mechanics” of solutions, which means how to use the **LC** and **DC** programs. Historical background is in Wilson (1994a) and Wilson (2001) while intuitive ideas are discussed in Wilson (1994b).

## 2.1. Does Light Curve Computation Require Correct Absolute Dimensions?

Persons familiar with old program versions will notice a (usually subtle) response of light curve output to changes in absolute masses and dimensional scaling in the **LC** and **DC** versions of 2003 and later. In versions of 1998 and earlier, one could enter any period or orbit size without affecting light curves, as the scaling of observable light (output) from luminosities (input) involved temperature but not  $\log g$  or chemical composition. That was because the old stellar atmosphere routine applied only to main sequence stars of normal composition, but now  $\log g$  and composition do affect light curve output, and the programs essentially compute  $\log g$  from  $GM/R^2$  (really from local conditions, including rotation and the other star’s gravity) with  $M$  and  $R$  dependent on period and absolute size. In old program versions one could even enter grossly wrong  $[P, a]$ ’s without adverse light curve effects, and some users may have become accustomed to that circumstance. However correct  $P$ ,  $a$ , and  $[M/H]$ ’s should be used with the new radiative treatment. Actually, the attendant effects are small, barring radically wrong values, but one should be consistent. Where orbit size is unknown, make a best guess rather than entering unrealistic numbers. In a non-simultaneous light-velocity solution, be sure that the final semi-major axis ( $a$ ) from the velocities is the same  $a$  used for the light curves. That condition will be satisfied automatically in a simultaneous solution. **LC** and **DC** are made to be mutually consistent but will have different  $L_2$ ’s and light if absolute dimensions and masses differ between the two programs. Naturally the foregoing warnings do not apply for blackbody computations, where the programs’ light curves are unaffected by absolute masses and dimensions.

## 2.2. Getting Started

Getting started should only be a matter of running the programs with the supplied sample input data and changing the numbers to those of particular binaries. Most persons will want to begin with a quick graphical fit, so as to be somewhere near to a proper solution. The following discussion specifically concerns light curves (MPAGE=1 in **LC**), although radial velocities (MPAGE=2) would be fitted in much the same way. It is best to be ready to feed either column 5 (light in program units) or column 6 (normalized light) to the graph. Remember that the normalized light column has no counterpart in **DC**, so after a few first cuts with normalized light (to reach rough agreement with the observations) switch from use of column 6 to use of column 5. At this point one should effect a correct transfer such that the column 5 numbers will be approximately the same as the column 6 (normalized) numbers that have been made to fit the light curve(s). This little problem involves the estimation of a scaling constant, or sometimes more than one. Please consult §4, p. 9, **Luminosity vs. Light (Flux)**, if you are not conversant with the distinction between “light” or “flux” on the one hand (an output quantity), and “luminosity” on the other (an input quantity). This will sound trickier than it is, but do not be concerned with making a perfect transfer from column 6 to column 5 output because you are iterating anyway in **DC** and only need to get close. Remember that the light of each star scales with its luminosity, while third light ( $\ell_3$ ) is just a direct add-on. Also remember that in all modes of program operation except modes 0 and  $-1$ , the luminosity of star 2 ( $L_2$ ) is computed by the program and scales with that of star 1 ( $L_1$ ), assuming fixed values for all other parameters. Therefore, the main, or perhaps only, scaling parameter in most situations is  $L_1$ . The simplest case is in mode 1, 2, 3, 4, 5, or 6 with no third light. Then, for example, doubling  $L_1$  will double column 5 output light. If there is third light, that also should be doubled if column 5 light ( $\ell_1 + \ell_2 + \ell_3$ ) is supposed to double. In mode 0,  $L_2$  does not scale with  $L_1$ , so  $L_2$  also would then need to be doubled. In mode  $-1$ ,  $L_1$  has only minor influence (through reflection heating), so  $L_2$  and  $\ell_3$  mostly control the scaling, although  $L_1$  also needs to be scaled in the general case. So to make column 5 agree with column 6, bump  $L_1$  and  $\ell_3$  (most modes) or  $L_1$ ,  $L_2$ , and  $\ell_3$  (modes 0 and  $-1$ ) up or down by a fixed factor.

With normalized output the idea is to select some phase where the synthesized curve is made to match the observed curve so that one can concentrate on form rather than the scale. Usually one chooses some innocuous phase outside eclipse where the brightness is not changing very fast. Vertical shifts of the whole normalized curve are made by changing input quantity **FACTOR**, if working in direct light, or **MZERO** if working in magnitudes. After you switch to column 5 output (getting ready for **DC**), control of vertical scaling passes to  $L_1$ ,  $L_2$ , and  $\ell_3$ , as explained above. After gaining some experience with **LC**, you will be using the normalized output only for a few preliminary runs and staying with program-unit light thereafter.

To start running **DC**, begin with the sample data, which contain control integers, initial parameter estimates, radial velocity curves for stars 1 and 2, and light curves in three bands. Run **DC** and inspect the output to be sure you understand everything. You may want to change a few things in the input to verify that the output responds as expected.

To begin a solution with your own data, change the sample input numbers to those appropriate to the observed binary, being careful to keep the original format. By this time you should have made a reasonable number of fitting experiments with **LC**, decided on the mode of program operation, decided on the parameter set and subsets to be adjusted, and estimated the **SIGMA**'s (standard deviations) of the observed curves (see below). You should also give a reasonable amount of thought to the sizes of the increments (**DEL**'s) used in forming the numerical derivatives. It is easy to fall into the error of trusting the **DEL**'s in the sample data to be appropriate for your binary, and sometimes they will be, but you cannot rely on that assumption. For example, suppose the mass ratio **DEL** is 0.01 and you just leave that value in for two binaries with mass ratios of 5.00 and 0.10. The increment for the first binary is one part in 500 (probably too small), while that for the second is one part in 10 (grossly too large, of course). Also remember not to try star spots that are unduly small in angular radius, because there are no fractional areas to the spots – they have staircase-like boundaries where their idealized circular outlines encounter the finite grid elements. Of course, the roughness of this simple treatment becomes particularly troublesome for small spots, and very small spots might not show up at all. Differential correction of spot parameters requires fine grids and considerable patience.

**DC** writes a table of numbers that are useful for estimating the **SIGMA**'s (standard deviations =  $\sigma$ 's) of the 1 or 2 velocity curves and **NLC** light curves. The table is labeled “*Sums of squares of residuals for separate curves, including only individual weights*”, which is exactly what is listed (neither curve-dependent nor level-dependent weights are included in the weights applied at this point). The sums of squares are just listed in the order their curves occur in the input. To form  $\sigma$ 's, calculate

$$\sigma = \sqrt{\frac{\Sigma W r^2}{n - m}},$$

where  $n$  is the number of data points in a curve and  $m$  is the number of adjusted parameters in the subset of primary interest. Usually  $n$  is much larger than  $m$ , so that the actual  $m$  makes little difference. The  $\sigma$ 's are used by **DC** to calculate curve-dependent weights – that is, to control the relative influences of the various curves. One can enter rough guesses for the  $\sigma$ 's in the first few iterations, then calculate them properly for the middle runs and fine-tuning runs. The rough guesses can be made by putting a straight line through a representative section of the data. **DC** can help in refining the estimated  $\sigma$ 's, as it provides for input of two phase ranges, within which it forms sums of squares of residuals, mean fluxes, and mean magnitudes. The start/stop phases are labeled “phase 1, phase 2, phase 3 and phase 4” in the output, and their FORTRAN names are **SPHAS1**, **SPHAS2**, **SPHAS3** and **SPHAS4**. With respective values of 0.07000, 0.43000, 0.57000, and 0.93000, they define the two phase ranges from 0°07000 to 0°43000 and 0°57000 to 0°93000. Usually it is best to choose a modest range for the “representative phases,” or perhaps an average of two such ranges. Often one would adopt an interval around one of the maxima or an average of the two maxima. Accordingly, the little “*Sums of squares ...*” table has sums of squares of residuals not only for each entire light curve, but also for a combined pair of phase ranges. The table also contains the reference flux and reference magnitude (for each band) that are averages over the two

chosen phase ranges. To have only one range count, enter a null range for the other one (such as 0.75 to 0.75). The reference flux is used to compute  $\sigma$  at a general light level, given  $\sigma_{\text{ref}}$  at the reference level. The relation is  $\sigma = \sigma_{\text{ref}}(\mathcal{F}/\mathcal{F}_{\text{ref}})^{n/2}$ , where  $\mathcal{F}$  is flux and  $n$  is the **NOISE** exponent described in §9.3, p. 26 and §15, p. 36. In the unlikely and easily prevented event that *no* points of a given light curve fall within either of the specified phase ranges, **DC** adopts the average for the entire phase range of 0.0000 to 1.0000 and prints a message to that effect, as a warning about the problem.

### 3. Modes of Program Operation (Solution Constraints)

Imagine that you know or are convinced of something about possible parameter values, and your “fact” is not in the form of a definite parameter value, but rather is a functional relationship among parameters. You do not claim to know the value of parameter  $c$ , but if someone tells you the values of parameters  $a$  and  $b$ , you then can compute  $c$ . The most common example is connected with one of the stars accurately filling a limiting lobe, as in a semi-detached case. It is impossible (or anyway, inadvisable) to say in less than a small monograph why we believe in the phenomenon of lobe filling and our ability to recognize semi-detached binaries as such. Of course, there are binaries for which we cannot be sure – the lobe may or may not be filled – but there are many for which we would “bet the ranch” that they are. In fact, for more than a few of the classical Algol type binaries, conflicts between mass ratios determined from radial velocities and mass ratios found from a lobe filling condition (through light curves) were eventually resolved in favor of lobe filling and light curves. S Cancri is a good example. For the simple case of a circular orbit and synchronous rotation, the mass ratio,  $q$ , of a semi-detached binary determines the relative size of the lobe filling star,  $R_{\text{lobe}}/a$ . Since the model expresses star size via the  $\Omega$  potential energy function,  $\Omega_{\text{lobe}}$  is determined by  $q$  and cannot be adjusted under a semi-detached or double contact constraint (modes 4, 5, 6). If we have lobe filling with non-synchronous rotation there is a functional relation among three parameters (now also  $F$  to express the rotation rate), and we add parameters  $e$  and  $\omega$  if the orbit is eccentric. In general, application of the lobe filling condition, or another condition, reduces the number of free parameters by one. Now because every parameter enlarges and complicates the parameter correlation matrix and thus weakens the solution, we should take advantage of every opportunity to eliminate free parameters. Essentially we thereby introduce information from sources external to the light and velocity curves, and we tell the Least Squares program “here is something about the binary that you cannot figure out from the data, but we are letting you know so that you can eliminate a whole dimension of incorrect solutions.” Each such condition can be represented as a constraint among possible parameter values, and a constraint or set of constraints is identified with a particular **operation mode** of **LC** and **DC**. Parameters that are set by constraints are no longer free parameters. The program computes them from the constraint relations and ignores their input values. Of course, they cannot be adjusted. Unfortunately, a few authors have failed to understand the idea of constrained solutions and have made comments something like “our program allows each star to have the radius that gives the best fit to the light curve, so that application of a

lobe filling constraint is unnecessary.” This is akin to saying “we amputate the legs so as to make walking unnecessary.”

**Mode -1:** This mode is for X-ray binaries for which the eclipse duration of a compact object is known from X-ray observations. The compact star must be star 1. The applied constraint is that the surface potential of the ordinary star (star 2) must be such as to produce the observed X-ray eclipse duration. Therefore, the program computes the  $\Omega_2$  potential from the input  $q$ ,  $F_2$ ,  $e$ ,  $\omega$ ,  $i$ , and X-ray eclipse semi-duration,  $\phi_e$  (FORTRAN name is **THE**). Note that solutions are not limited to circular orbits or synchronous rotation – any  $e$  between 0 and 1 is allowed, as is any  $F_2$ . Note also that lobe filling is not implied. It is recommended to use equal temperatures for the two stars in most situations where one would apply mode -1 logic. With component 1 a neutron star, for example, it will be unrealistic to deal with the actual surface temperature (which usually will be in a bizarre distribution – perhaps 50,000,000 K in a small hot spot or two and far lower over the rest of the surface. The tiny ultra hot surface will contribute negligible optical flux anyway. Usually it will be best to assign star 1 a temperature equal to that of star 2, so as to be sure of having zero bolometric correction in the reflection effect, thereby keeping things relatively simple.

**Mode 0:** No constraints are applied in mode 0, and the component luminosity ratio is not even required to be consistent with the surface temperatures. That is, the program uses  $L_1$  and  $L_2$  as supplied, and does not recompute  $L_2$  based on the input temperatures.

**Mode 1:** This is a mode for overcontact binaries, such as W UMa stars. Six or seven constraints are applied, depending on whether fixed or computed limb darkening is specified (viz. the end of §9.1, p. 22). The first is that the surface potential of star 2 is equal to that of star 1 ( $\Omega_2 = \Omega_1$ ). Another is that the polar temperature of star 2 is set by the gravity brightening law of the entire common envelope, which is done via Eqn. 8 of Wilson (1979). Other constraints are that the gravity brightening, bolometric albedo, and perhaps limb darkening parameters of stars 1 and 2, are the same.<sup>2</sup> Finally, the luminosity of star 2 is computed from the other parameters via blackbody or stellar atmosphere radiation formulas. Therefore, parameters  $\Omega_2$ ,  $g_2$ ,  $T_2$ ,  $A_2$ ,  $L_2$ ,  $x_2$ , and  $y_2$  are not free in mode 1. A consequence of mode 1 operation is that there is a smooth variation of surface brightness over the entire common envelope (except at spot edges) with no discontinuity where the stars join together and thus one continuous gravity brightening law. In mode 1 light curve fitting experiments, there will be essentially no freedom to change the relative primary and secondary eclipse depths, which will be set almost entirely by geometry (although a little by gravity brightening, limb darkening, etc.).

**Mode 2:** This mode is for detached binaries with no constraints on the potentials. The only constraint is that the secondary luminosity,  $L_2$ , is computed from the other parameters via the specified radiation prescription (blackbody or stellar atmosphere). That is,  $L_2$  is coupled to the

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<sup>2</sup>Naturally the  $x_2 = x_1$  and  $y_2 = y_1$  conditions are applied only for fixed  $x, y$  coefficients, not for actively computed  $x, y$ 's.

temperatures. The temperature-luminosity coupling can be severed by setting control integer IPB to 1 (normal value = 0). Mode 2 is the same as mode 0 except for the one constraint, and is exactly like mode 0 if IPB is set to 1.

**Mode 3:** This mode is for overcontact binaries and is like mode 1 except that the constraints on  $g_2$ ,  $T_2$ ,  $A_2$ ,  $x_2$ , and  $y_2$  are not applied. The other two mode 1 constraints (on  $\Omega_1$  and  $L_2$ ) are applied in mode 3, but  $T_2$ ,  $A_2$ ,  $g_2$ ,  $x_2$ , and  $y_2$  are free. *This treatment is changed from the 1998 version* (see §20, p. 43). The stars can be overcontact in mode 3, yet have much different surface brightnesses. In structural language, they can be in geometrical contact without being in thermal contact.

**Mode 4:** This mode is for semi-detached binaries with star 1 accurately filling its limiting lobe, which is the classical Roche lobe for synchronous rotation and a circular orbit, but is different from the ordinary Roche lobe for non-synchronous and eccentric cases. The applied constraints are that  $\Omega_1$  has the lobe filling value and that  $L_2$  is coupled to the temperatures (unless IPB=1).

**Mode 5:** The same as mode 4, except that it is star 2 that fills its limiting lobe. This is the usual mode for Algol-type binaries.

**Mode 6:** This mode is for double contact binaries, in which both stars accurately fill their limiting lobes (Wilson 1979). Astrophysically this makes sense only if at least one of the stars rotates non-synchronously. Mode 6 applies the surface potential constraints of modes 4 and 5 together.

**Note on lobe filling:** The program operates under a general definition of a limiting lobe that applies for non-synchronous as well as synchronous rotation and for eccentric as well as circular orbits. The classical Roche lobe is a special case that obtains for the synchronous, circular orbit case. In the general definition, *a limiting lobe is an equipotential for which the effective gravity is zero on the line of star centers at periastron* (Wilson 1979). In a strict sense, equipotentials do not exist in the eccentric orbit case, but in essence the concept of equipotentials still is useful in most realistic situations (Avni 1976; Wilson 1979).

#### 4. Luminosity vs. Light (Flux)

It is essential to emphasize the very important, yet commonly ignored, distinction between “light” or “flux” on the one hand, and luminosity on the other. As simple as this point is, it has led to remarkable confusion in the literature. The problem has its roots in models of past decades that could have made the distinction but did not, and it has carried over into the general binary star literature of today. A luminosity as discussed here is characterized by a bandpass and bandpass properties (sometimes written “band” for brevity). Where it is necessary to refer to a bolometric luminosity, the qualifier “bolometric” can specifically be attached. In some papers, unqualified luminosity means bolometric luminosity, but in this document unqualified luminosity means bandpass luminosity.

A discussion of this point is in the description of subroutine **LUM** in Wilson (1993). The main thing

to remember is that luminosity is a global quantity, represented by a single number for each star and band. It does not depend on aspect (neither on inclination nor on phase). Some authors write of “normalizing luminosities by the sum of the luminosities outside eclipse.” However, luminosity has nothing to do with eclipses or their absence, nor with any other aspect-dependent variation. Flux (or, in more relaxed terminology, “light”) is a directly observable (aspect-dependent) quantity, while luminosity is a model parameter that cannot be observed directly but must be inferred as part of a photometric solution. The luminosities,  $L_1$  and  $L_2$ , still can be normalized by their sum (a common and reasonable practice) but two points need to be kept in mind. The first is that the normalization must be made only after completion of a solution, not during its progress. The reason is that the luminosities are the main handles for scaling of (output) flux. As such they need to float freely so that the model is able to match the observed fluxes, which often are in an arbitrary unit, since the measurements are commonly made with respect to the flux from a comparison star. The second point is that  $L_1$  and  $L_2$  are not the same kind of quantity as third light (a flux), are not to be compared directly with third light, and are not in the same units. This is important because many papers have listed values of third light with no indication of what the number means. Not only is the printed  $\ell_3$  thereby rendered useless, but strictly speaking the entire solution is made meaningless because  $\ell_3$  usually is significantly correlated with other parameters. An estimate of third luminosity, based on third light, involves an assumption about the direction dependence of the radiation of the third source. If that radiation is emitted isotropically, then third luminosity,  $L_3$ , will be  $4\pi$  times  $\ell_3$ , which is per  $1/4\pi$  of the area of a sphere centered on the binary (“a steradian’s worth of area”). That  $L_3$  value is to be compared with the un-normalized  $L_1$  and  $L_2$ . A meaningful way to specify the unit of a published  $\ell_3$  is to express it in the light (relative flux) of the multiple star system at a definite phase. For example, suppose the solution output light is 1.0500 at reference phase 0.2500 for a particular light curve. Then divide  $\ell_3$  and its error estimate by 1.0500, list those numbers, not the direct  $\ell_3$  program output, and tell the reference phase in a footnote to the table of results.

## 5. Circumstellar Light-Attenuating Regions

Light curves of some interacting binaries show effects of attenuation of star light by circumstellar matter. Most such matter is gaseous, although dust attenuation may be significant in a few cases. The attenuation may be due to Thomson scattering, Rayleigh scattering, another scattering process, or true absorption. Since circumstellar matter follows dynamical trajectories, one might expect there to be little interest in attenuating regions that are fixed in a coordinate frame that rotates with the binary, but such is not entirely the case. A few binaries (e.g. RZ Sct, AX Mon) have approximately stationary loci of circumstellar gas that distort their light curves. The loci may be stream-stream, stream-disk, or stream-wind interaction regions. Efforts to represent such light curve distortions via bright or dark star spots sometimes can rule out a spot explanation and point to essentially fixed attenuation regions (hereafter “clouds,” for brevity). The model includes  $n$  spherical semi-transparent clouds specified by their locations  $(x, y, z)$  [in a rectangular frame that co-rotates with

the stars], cloud radius ( $r$ ), density ( $\rho$ ), electron density ( $n_e$ ), and mean molecular weight per free electron ( $\mu_e$ ). The  $x$  coordinate is zero at the center of star 1 and increases toward star 2, reaching  $+D$  at the center of star 2. The  $x, y, z$  system is right handed and serves for the entire binary system. The part of the line of sight that passes through the various clouds is computed individually for the lines of sight to all surface points and individually for all clouds. Regions of variable density can be made by nesting individual clouds. Regions of non-spherical shape can be approximated by overlapping spherical clouds. Each cloud is allowed its own attenuation law, whose general form is

$$\frac{d\tau}{ds} = \sigma_e n_e + (\kappa_\lambda + \kappa_{sb}) \rho$$

where  $\tau$  is optical thickness,  $\sigma_e$  is the Thomson scattering cross section per electron,  $s$  is distance along the line of sight (in cm),  $\kappa_\lambda$  is a wavelength-dependent opacity, and  $\kappa_{sb}$  is an additional opacity for a specific band ( $\kappa$ 's are in  $\text{cm}^2/\text{g}$ ). The  $\kappa_{sb}$  term might represent, for example, opacity due to absorption lines averaged over a particular band. The  $\kappa_\lambda$  term is

$$\kappa_\lambda = \kappa_0 \lambda^\alpha$$

where  $\kappa_0$  and  $\alpha$  are input quantities. Each cloud has its individual  $\kappa_0$ ,  $\alpha$ , and  $\kappa_{sb}$ . However, to make it easy to change the  $\kappa_{sb}$ 's of all clouds together, the  $\kappa_{sb}$ 's are not entered directly as individual numbers. Instead one enters an overall  $\kappa_{sb}$  and the fraction of it that applies for each cloud. Thus

$$\kappa_{sb} = f_i \kappa_{sb0}$$

where all the  $f_i$  can be unity if  $\kappa_{sb}$  is to be the same for all clouds, or non-unity if  $\kappa_{sb}$  is to differ from cloud to cloud. The program figures absolute lengths from the system geometry, including the orbital semi-major axis.

At present the clouds only attenuate starlight that passes through them, but they may be made to scatter starlight toward the observer in a future program version.

## 6. Spectral Line Profiles

Profiles of absorption and emission lines are generated for MPAGE=3. The profiles are for rotation only, although other broadening mechanisms may be added later. Blending is incorporated, including blending of mixed absorption and emission lines. Lines can originate either from an entire star or from designated sub-areas of the surface, as explained below. Only **LC** (not **DC**) computes line profiles. Spectra are formed by binning, with the spectra of the two stars formed separately. The user can add them (weighted by observable flux) if spectra of the binary are needed. Computations for each star are characterized by four quantities related to spectral and accuracy characteristics (BINWM1, SC1, SL1, NF1 for star 1 and BINWM2, SC2, SL2, NF2 for star 2), that have the following meanings:



**BINWM1(2)**: The bin width in microns. Too small a bin width gives noisy profiles. Too large a bin width gives insufficient spectral resolution.

**SC1(2)**: The continuum scale (continuum flux at the reference wavelength). The unit is decided by the user.

**SL1(2)**: The continuum slope in flux units per micron.

**NF1(2)**: Grid fineness for micro-integration on each surface element. **NF1(2)=1** means that there is no micro-integration. **NF1(2)= $n$**  breaks each surface element into  $n^2$  pieces, each with its own radial velocity, thus improving integration accuracy.

With **MPAGE=3**, an **LC** input line with the above quantities is required for each star, even if one of the stars is assigned no spectral lines (see sample input file **lcin.dat3**). **DC** program input does not have the analogous data lines.

Following the star-dependent input are data for individual spectral lines in two sets (for star 1, then star 2). The quantities are (FORTRAN names):

**WLL1(2)**: The line rest wavelength in microns for a line of star 1 (or 2).

**EWID1(2)**: The line equivalent width, in microns – the traditional measure of line strength, for a line of star 1 (or 2). Absorption and emission lines both have positive equivalent width by program convention. Whether a line is in absorption or emission is controlled by parameters **DEPTH1** and **DEPTH2** (see next ¶).

**DEPTH1(2)**: Rectangular line depth for a line of star 1 (or 2). Line profiles are formed by binning of Doppler shifted elements that have rectangular profiles, each with a depth and a width. The user supplies the depth and the program then calculates the width needed to reproduce the specified equivalent width. The depth is relative to a unit continuum, so 0.80000 means that 80 percent of the continuum flux is missing within the rectangular profile element, or that the residual flux is 20 percent of the continuum. Negative depths correspond to emission lines, so  $-0.50000$  means 50 percent above the continuum. Depths must be less than 1.0000 (i.e. an absorption line cannot go to zero flux or below), but can be less than  $-1.0000$  (an emission line can go arbitrarily high).

**KKS**: This integer specifies a surface region associated with a given spectral line. If **KKS=0**, the line is not specific to a location but applies to the entire star. If **KKS=1**, then the line applies only to the first spot on that star; if **KKS=2** it applies only to the second spot, and so on. Naturally the star must have spots for this scheme to work, but the spots need not be hot or cool spots – they can have temperature factors of unity. Negative **KKS** specifies avoidance of regions. Thus **KKS=-4** means that the spectral line applies everywhere on the star except within spot 4. If you find this confusing, just set **KKS=0** and the line applies in the old simple way – everywhere on the star.

If computed absorption lines have flat bottoms, you probably used too small a depth. What happens is that the computed line is a superposition of many elemental bar-like lines, so if the bars are not deep, then they have to be wide in order to have the specified equivalent width. So essentially

the problem is one of **DEPTH** being too small for a given equivalent width. The “flat-bottom” problem may be especially troublesome for slowly rotating stars. The converse problem is where the equivalent width is very small and **DEPTH** is large (like 0.99). The elemental bars are then very narrow (needle-like). This situation can produce raggedy lines, especially for fast rotating stars. The remedy is to use a smaller **DEPTH**.

## 7. Model Parameters

These are the same for the **LC** and **DC** programs, although not all of the parameters are not among the 40 that can be adjusted by the present version of **DC**. The actual number of parameters subject to adjustment is greater than 40 for two reasons. First, although **DC** can adjust the parameters of at most two star spots in any one run (iteration), successive iterations can adjust the parameters of other spots (viz. definitions of **KSPA**, **KSPB**, **NSPA** and **NSPB** in §9.3, pp. 24-24). Second, curve-dependent parameters (limb darkening coefficients, relative bandpass luminosities, third light) have  $n$  values for  $n$  simultaneously fitted light curves. For example, specifying the adjustment of  $\ell_3$  in a simultaneous solution of four light curves will produce four  $\ell_3$  corrections, one for each curve. The word “curve” is used here in the sense of “light curve” or “radial velocity curve,” etc. Parameters are identified below according to their FORTRAN names.

### 7.1. Curve-independent parameters that cannot be adjusted by DC

**THE**: ( $\phi_e$ ) The semi-duration of primary eclipse in phase units (i.e. range 0 to 1 for a cycle). This parameter is used only in mode  $-1$  and usually relates to binaries in which star 1 has negligible size compared to star 2. The idea is to fix  $\phi_e$  according to X-ray eclipses of a neutron star, black hole, or white dwarf, and require the overall solution to be consistent with that value. Although mode  $-1$  logic was originally intended for X-ray eclipses of compact objects, it also will work for optical eclipses of any star for which  $R_1/R_2$  is extremely small, such as a main sequence star with a red giant companion. The orbit can be eccentric and rotation can be non-synchronous. Parameter  $\phi_e$  is ignored in all other operation modes. See the explanation of mode  $-1$  on p. 8 in §3. The mathematics for the general case with any eccentricity and any rotation is in Wilson (1979).

**XBOL1**, **XBOL2** ( $x_{bol1}, x_{bol2}$ ): The coefficients of  $\cos \gamma$  in the bolometric limb darkening law. They are used only in computation of “detailed” reflection (Wilson 1990), with **MREF**=2. See  $y_{bol1}$ ,  $y_{bol2}$  below for the complete law and explanation.

**YBOL1**, **YBOL2** ( $y_{bol1}, y_{bol2}$ ): If control integer **LD1**(2)=+2 or  $-2$ , these are the coefficients of the  $\cos \gamma \ln(\cos \gamma)$  term in the bolometric logarithmic limb darkening law. The complete logarithmic law is

$$\frac{I}{I_0} = 1 - x + x \cos \gamma - y \cos \gamma \ln(\cos \gamma),$$

which was advocated by Klinglesmith & Sobieski (1970). If LD1(2)=+3 or -3, they are the coefficients of the bolometric square root law. The complete square root law (Diaz-Cordovés & Giménez 1992) is

$$\frac{I}{I_0} = 1 - x + x \cos \gamma - y (1 - \sqrt{\cos \gamma}).$$

Coefficients for all these darkening laws have been tabulated by Van Hamme (1993).

XCL, YCL, ZCL: Rectangular coordinates of the centers of spherical circumstellar clouds (see §5, p. 10).

RCL: Radii of individual circumstellar spherical attenuating regions in unit of  $a$ .

EDENS ( $n_e$ ): Electron densities in  $\text{cm}^{-3}$  for individual attenuating clouds. For a given cloud,  $n_e$  is constant, although clouds can be nested or overlapped.

XMUE ( $\mu_e$ ): Mean molecular weight in atomic mass units per free electron for individual attenuating clouds, and constant throughout a given cloud.

ENCL ( $\alpha$ ): Exponent in the wavelength-dependent term of the attenuation law for individual attenuating clouds (see p. 10, §5). The program internally computes densities,  $\rho$ , from  $n_e$  and  $\mu_e$ .

## 7.2. Curve-dependent parameters that cannot be adjusted by DC

WLA: The observational wavelengths in microns of each light or velocity curve. Wavelengths need to be entered for velocity curves, although they have little effect on the output, and any wavelength somewhere near the spectral region of interest should be adequate. Wavelengths are no longer used for light curves, which now are based on integrated bandpass radiation (see pages 30–31, §12). Wavelengths are still entered for use as reference wavelengths for line profiles and for opacity computations in circumstellar attenuation.

Y1A, Y2A ( $y_1, y_2$ ): These are the bandpass-specific limb darkening coefficients *in the non-linear terms*. The laws have the same forms as for bolometric limb darkening (p. 13). In the case of definite (i.e. fixed, not re-computed) limb darkening, there is one value for each of these coefficients for each light or velocity curve or set of line profiles. **LC** uses both wavelength and bandpass input for line profile computation (wavelength for basic profiles, bandpass for weighting by flux). Enter integer parameter LD1(2)=+1 or -1 for the linear cosine law (in which case  $y_1$  and  $y_2$  are assumed to be zero), LD1(2)=+2 or -2 for the logarithmic law, or LD1(2)=+3 or -3 for the square root law. Negative LD1(2) is for internally computed  $x, y$ 's and positive LD1(2) for fixed  $x, y$ 's. Parameters  $y_1$  and  $y_2$  cannot be adjusted by **DC**.

### 7.3. Curve-independent parameters that can be adjusted by DC

HJD0 ( $t_0$ ): This is the zero point of the orbital ephemeris. Usually one uses Heliocentric Julian Date, although that is only a convention and any consistent system of time can be used.

PERIOD ( $P_0$ ): The binary orbit period at the reference time  $t_0$ , ordinarily in mean solar days.  $P_0$  affects radial velocity amplitudes and is used to compute phase from time (if JDPHS=1) and time from phase (if JDPHS=2).  $P_0$  can be adjusted only if JDPHS=1 and observation times (rather than phases) are entered.

DPDT ( $\frac{dP}{dt}$ ): The first time derivative of the orbital period. Second and higher derivatives are not used in the present program. DPDT can be adjusted only if JDPHS=1 and observation times (rather than phases) are entered. This quantity is dimensionless.

E ( $e$ ): Binary orbital eccentricity.

PERR0 ( $\omega_0$ ): Initial argument of periastron **for star 1**. The argument of periastron for star 2 differs by  $\pi$  radians. PERR0 is  $\omega$  at the reference time of the ephemeris,  $t_0$ . The program is written so that, as the argument of periastron changes, both eclipses have excursions in phase in the same way as a real binary. For circular orbits, the program ignores the input value and sets  $\omega$  to  $\pi/2$  radians.

DPERDT ( $\frac{d\omega}{dt}$ ): The first time derivative of  $\omega$ . DPERDT can be adjusted only if JDPHS=1 and observation times (rather than phases) are entered. The instantaneous argument of periastron is  $\omega = \omega_0 + \frac{d\omega}{dt}(t - t_0)$ . The program does not consider any more complicated variations of  $\omega$ . The unit is radians per adopted time unit (usually mean solar day).

A ( $a$ ): The length of the semi-major axis in solar radii ( $6.96265 \times 10^5$  km) of the relative orbital ellipse. It is the sum of the two absolute semi-major axes, so  $a = a_1 + a_2$ .

F1, F2 ( $F_1, F_2$ ): The ratio of the (constant) axial rotation rate to the mean orbital rate for stars 1 and 2, respectively. The angular rotation is assumed to be uniform (not latitude dependent). Value unity represents synchronous rotation in a circular orbit. In eccentric cases it is expected that rotation will tend to synchronize to the periastron angular rate because of the strong dependence of the tide raising force on distance. Periastron-synchronized  $F$  is given by

$$F = \sqrt{\frac{1+e}{(1-e)^3}}.$$

The  $F$ 's affect star figures, surface brightnesses (through the gravity effect), limiting lobe sizes, and thus both light curves and radial velocities.

VGA ( $V_\gamma$ ): The radial velocity of the (possibly multiple-system) barycenter in the unit VUNIT km/s.  $V_\gamma$  is assumed constant, although the EB center of mass radial velocity follows from motion about the barycenter if there is a third body.

PSHIFT ( $\phi_0$ ): A constant shift applied to computed phases. Usually one enters +0.0000 for  $\phi_0$ ,

but it may be convenient to shift the phases. For example, a phase shift of about half a cycle can effectively interchange the star labels (1 vs. 2) without altering the observational data. The main purpose of  $\phi_0$  is to allow the **DC** program to adjust for a zero point error in the ephemeris used to compute the phases. The unit is the orbital period. One should not adjust both **PSHIFT** and **HJD0** because they will be perfectly correlated.

**XINCL** ( $i$ ): The binary orbital inclination to the plane of the sky, in degrees. If the inclination is in the range 0 to  $90^\circ$ , the binary orbits counter-clockwise as projected onto the plane of the sky, while above  $90^\circ$  it orbits clockwise, according to the program's coordinate conventions. Those conventions were different prior to the revision of 1992.

**GR1, GR2** ( $g_1, g_2$ ): The exponents in the bolometric gravity brightening (a.k.a. darkening) law for stars 1 and 2, respectively. Value 1.000 means that bolometric flux is proportional to local effective gravity, while 0.000 means that it is constant over the surface (ignoring other effects such as spots, reflection heating, etc.). The  $g$ 's are expected to be unity for radiative envelopes, while they should be smaller for convective envelopes, perhaps about 0.3. Some other programs use a gravity brightening exponent expressed in terms of effective temperature, for which the usual symbol is  $\beta$ . The quantities are related by  $g = 4\beta$ .

**TAVH, TAVC** ( $T_1, T_2$ ): The mean surface effective temperatures of stars 1 and 2, respectively, not including re-radiation (reflection) or spots. The mean is weighted by the local bolometric flux. The program accepts  $T_1$  and  $T_2$  as model parameters and converts to local surface temperatures for internal computations. The conversion between mean and polar temperatures is made via Eqn. 8 of Wilson (1979), and the local surface temperatures are then computed from the polar temperatures and the gravity brightening law. The unit is 10,000 K.

**ALB1, ALB2** ( $A_1, A_2$ ): The bolometric albedos for reflection heating and re-radiation on stars 1 and 2, respectively. The bolometric albedo is the local ratio of re-radiated bolometric energy to received bolometric energy. It is assumed to be constant for each star. The expected value for radiative envelopes is 1.0, while for convective envelopes it should be perhaps 0.5, although observations sometimes indicate values between 0.5 and 1.0.

**PHSV, PCSV** ( $\Omega_1, \Omega_2$ ): These are the "potentials" for stars 1 and 2, respectively, that originally were defined by Kopal (1954) for the synchronous, circular orbit case. They would be actual potentials, except that a term was deleted in Kopal's convention. The deleted term depends on mass ratio but not on position, so  $\Omega$  gradients are equivalent to potential gradients. Fixed  $\Omega$  specifies a constant potential energy (gravitational plus rotational) over the surface of each star. A generalized defining equation (Eqn. 1 of Wilson 1979), based on contributions by Plavec (1958), Limber (1963), and Avni (1976), allows the  $\Omega$ 's to serve also for non-synchronous rotation and eccentric orbits. Together with the mass ratio, rotation rate, orbital eccentricity, argument of periastron, and phase, the  $\Omega$ 's specify the size, figure, surface gravity, and certain other geometric properties of the stars. Special values of the  $\Omega$ 's correspond to exact filling of limiting lobes.

**RM** ( $q$ ): The mass ratio of stars 2 and 1,  $m_2/m_1$ .

A3B ( $a_{3b}$ ): The length of the semi-major axis of the relative orbit of the EB center of mass around the system’s barycenter, in solar radii.

P3B ( $P_{3b}$ ): The third body orbit period in mean solar days.

XINC3B ( $i_{3b}$ ): The third body orbital inclination to the plane of the sky, in degrees. This parameter should *not* be adjusted if  $a_{3b}$  is adjusted since  $a_{3b}$  and  $i_{3b}$  are perfectly correlated.

E3B ( $e_{3b}$ ): Third body orbital eccentricity.

PERR3B ( $\omega_{3b}$ ): Argument of periastron of third body orbit.

TC3B ( $t_{c3b}$ ): Time of superior conjunction of the EB center of mass with respect to the third body, usually in HJD.

XLAT: The “latitude” of a star spot center, measured from 0 radians at the “north” (+ $z$ ) pole to  $\pi$  radians at the “south” pole. XLAT’s are subscripted by star (1 or 2) and by spot number on a star (viz. explanation of KSPA, NSPA, KSPB and NSPB in §9.3, pp. 24-24).

XLONG: The longitude of a star spot center, measured counter-clockwise (as viewed from above the + $z$  axis) from the line of star centers from 0 to  $2\pi$  radians. XLONG is subscripted in the same way as XLAT.

RADSP: The angular radius of a star spot, in radians. The angle is subtended by the spot radius at the center of the star. RADSP is subscripted in the same way as XLAT.

TEMSP: The “temperature factor” of a spot that specifies the ratio of local spot temperature to local temperature that would obtain without the spot. A TEMSP larger or smaller than unity corresponds to the spot being hotter or cooler than the un-spotted surface, respectively. TEMSP is constant for a given spot, but local temperature will vary over a spot if the underlying un-spotted temperature varies. TEMSP is subscripted in the same way as XLAT.

#### 7.4. Curve-dependent parameters that can be adjusted by DC

HLA, CLA ( $L_1, L_2$ ): Bandpass luminosities for stars 1 and 2, respectively. There has been some confusion about the units of luminosity and of light, which is partly due to a tradition in the binary star field of normalizing luminosity and light separately, and failing to recognize that they are fundamentally distinct (although connected) quantities (viz. §4, p. 9). The program luminosity unit is effectively user-supplied, and determines the unit of output light (i.e.  $\ell_1$  or  $\ell_2$  bandpass flux). The output flux will integrate to the luminosity over a sphere at any large distance, centered on the binary system. The computed fluxes will be in the unit  $1/4\pi$  luminosity units/(steradian’s worth of area). Now this may not sound like correct flux units – with “steradian” in there it sounds like intensity. However, that last item, “steradian’s worth of area” is indeed an area, not a solid angle. So to be formally correct, replace “steradian’s worth of area” with “ $d^2 \text{ cm}^2$ ,” where  $d$  is the

assumed binary–observer distance in cm. Pictorially, imagine the observer’s detecting instrument covering 1 steradian (a little on the big side, but one can always re-scale to more practical units). To keep things simple, imagine that the star radiates isotropically and that we enter a luminosity of  $4\pi$ , in our chosen unit, for one of the stars. The program will produce an output flux of 1.000 for that star for all phases and inclinations (outside eclipse), and since that flux refers to 1 steradian, the  $4\pi$  steradians surrounding the star would have  $4\pi$  units, which is just what we entered as the luminosity. If we specify the luminosity unit to be, say,  $1 \times 10^{33}$  erg/sec/micron, then the luminosity is  $4\pi \times 10^{33}$  erg/sec/micron, and the flux (light) is  $1 \times 10^{33}$  erg/sec/micron/ $d^2$  cm<sup>2</sup> (constant in this special case).

**X1A, X2A** ( $x_1, x_2$ ): These are the wavelength-specific limb darkening coefficients in the linear terms. The laws have the same forms as for bolometric limb darkening (p. 13). Parameters  $x_1$  and  $x_2$  are adjustable only if the input darkening coefficients are specified as being definite (i.e. not to be re-computed), which is decided by input quantities **LD1** and **LD2** being positive (+). There is one  $x_1$  and one  $x_2$  value for each light or velocity curve or set of line profiles. For negative **LD1** or **LD2**, local surface computation of limb darkening coefficients as functions of  $T_{eff}$ ,  $\log g$ , and  $[M/H]$  is done by subroutine **LIMDARK**, which is based on the Van Hamme (1993) limb darkening tables. Flux-weighted mean  $x, y$ ’s for both stars are printed by **LC** if **MPAGE=1**.

**EL3A** ( $\ell_3$ ): Third light. There is one value for each light curve, but of course no value for a radial velocity curve. The unit should be the total system light at a specified phase. For example, suppose  $\ell_3$  (program input-output value) for some particular light curve is 0.0500, the specified phase is chosen to be 0.2500, and the total system light produced by **LC** at phase 0.2500 is 1.0400. Then the number to be published for the  $\ell_3$  of that curve would be 0.0500 divided by 1.0400, or 0.0481. The standard error printed by **DC** for  $\ell_3$  also should be divided by 1.0400 for publication. A footnote can be placed in the publication to tell the reference phase.

## 7.5. Ephemeris Solutions and Third Body Parameters

Ephemeris parameters  $T_0$ ,  $P_0$ , and  $dP/dt$  can be found from whole light and velocity curves by **DC**, as an alternative to ephemerides from eclipse timing diagrams. The simple but necessary mathematics for rigorous conversion from time to phase and phase to time is in (Wilson 2005) along with a history of its use in **LC** and **DC**, and with citations to a number of application papers. WD’s 2007 version also can extract third body (3b) orbit parameters. These are parameters numbered 30 to 35 in §8, p. 20. Pertinent observational phenomena are light-time and radial velocity shifts (not third light,  $\ell_3$ ). Analysis of light and velocity shifts and third light effects may be done together, with the two kinds of results being essentially independent. Relevant history, strategies, and logic of the 3b development and a similar one by Hadrava (2004) are in Van Hamme & Wilson (2007), as are the forms of 3b derivatives required by **DC**. Rather than repeat here the experience conveyed in Van Hamme & Wilson (2007), we recommend that potential users of **DC**’s 3b capability read that paper. A solution that includes 3b parameters is basically the same as other solutions, except that

(1) the increased number of parameters may make application of the MMS imperative (see §10.1, p. 27) and (2) a satisfactory starting estimate for  $P_{3b}$  may require separate use of power spectral analysis. Simultaneous processing of light and velocity curve data is an important advantage of **DC**'s 3b facility, as the two types taken together reduce the severity of the well known data gap problem for period searches. However, even with that advantage, initial application of power spectral analysis often will be needed to sift the correct period from apparent periods that are only artifacts of the timewise observing gaps. Note the example of VV Orionis, where recent papers find no evidence for a supposedly well established 3b period (Terrell, Munari, & Siviero 2007; Van Hamme & Wilson 2007).

## 8. Parameter Order

The **DC** program now has 60 parameter channels, 40 of which are assigned to specific parameters while the other 20 are for future expansion. The 40 activated **DC** parameter channels are assigned as follows:

- (1) - Spot 1 latitude in radians
- (2) - Spot 1 longitude in radians
- (3) - Spot 1 angular radius in radians
- (4) - Spot 1 dimensionless temperature factor
- (5) - Spot 2 latitude in radians
- (6) - Spot 2 longitude in radians
- (7) - Spot 2 angular radius in radians
- (8) - Spot 2 dimensionless temperature factor
- (9) - Orbital semi-major axis in solar radii,  $a = a_1 + a_2$
- (10) - Orbital eccentricity,  $e$
- (11) - Initial argument of periastron in radians,  $\omega_0$
- (12) - Dimensionless rotation parameter for star 1,  $F_1$
- (13) - Dimensionless rotation parameter for star 2,  $F_2$
- (14) - Phase shift = phase of primary conjunction (for  $\omega = \pi/2$ ),  $\phi_0$
- (15) - System center of mass radial velocity (systemic velocity),  $V_\gamma$
- (16) - Orbital inclination in degrees,  $i$



- (17) - Gravity law exponent for star 1,  $g_1$
- (18) - Gravity law exponent for star 2,  $g_2$
- (19) - Mean surface temperature of star 1 in 10,000 K,  $T_1$
- (20) - Mean surface temperature of star 2 in 10,000 K,  $T_2$
- (21) - Bolometric albedo of star 1 (dimensionless),  $A_1$
- (22) - Bolometric albedo of star 2 (dimensionless),  $A_2$
- (23) - Modified dimensionless surface potential of star 1,  $\Omega_1$
- (24) - Modified dimensionless surface potential of star 2,  $\Omega_2$
- (25) - Mass ratio,  $q = m_2/m_1$
- (26) - Reference epoch in ephemeris (conventionally in Heliocentric JD),  $t_0$
- (27) - Orbital period at reference epoch,  $P_0$
- (28) - First time derivative of the orbital period,  $dP/dt$
- (29) - First time derivative of the argument of periastron,  $d\omega/dt$
- (30) - Semi-major axis length of third body orbit in solar radii,  $a_{3b}$
- (31) - Period of third body orbit in days,  $P_{3b}$
- (32) - Inclination of third body orbit to the plane of the sky in degrees,  $i_{3b}$
- (33) - Eccentricity of third body orbit,  $e_{3b}$
- (34) - Argument of periastron of third body orbit in radians,  $\omega_{3b}$
- (35) - Time (conventionally Heliocentric Julian Date) of superior conjunction of the EB center of mass with respect to the third body,  $T_{c_{3b}}$
- (36 to 55) - Not used, reserved for future expansion.
- (56) - Bandpass luminosity of star 1,  $L_1$
- (57) - Bandpass luminosity of star 2,  $L_2$
- (58) - Bandpass linear limb darkening coefficient for star 1,  $x_1$
- (59) - Bandpass linear limb darkening coefficient for star 2,  $x_2$
- (60) - Bandpass third light,  $\ell_3$

## 9. Control Integers, Units, Scaling Factors, Special Quantities

These numbers control program operation and are assigned according to the aims of the user. They are given here by their FORTRAN names.

### 9.1. Those Common to LC and DC

**JDPHS:** This is 1 if the independent variable is time and 2 if it is phase. Setting **JDPHS**=1 in **LC** causes time (ordinarily Heliocentric Julian Date) to be stepped from **HJDST** to **HJDSP** in uniform intervals of length **HJDIN** (see below). Phases for those times are computed according to the supplied ephemeris, including the effect of  $dP/dt$ . Setting **JDPHS**=2 in **LC** causes phase to be stepped from **PHSTRT** to **PHSTOP** in uniform intervals of length **PHIN**. Times for those phases are computed according to the supplied ephemeris, again including the effect of  $dP/dt$ . Regardless of whether **JDPHS** is 1 or 2, mutually consistent time and phase are listed in the output (columns 1 and 2, respectively). At present, the ephemeris includes only the initial epoch ( $t_0$ ), the period at that epoch ( $P_0$ ), and the first time derivative of the period,  $dP/dt$ .

**MODE:** This integer can be  $-1$ ,  $0$ ,  $1$ ,  $2$ ,  $3$ ,  $4$ ,  $5$ , or  $6$  according to the constraints or lack of constraints to be applied. The operation modes are described in Wilson (1993) and on pages 7–9, §3.

**IPB:** Assign **IPB**=0 for normal **MODE** 1, 2, 3, 4, 5, or 6 operation in which star 2's luminosity ( $L_2$ ) is to be computed from temperatures  $T_1$  and  $T_2$ , the luminosity of star 1, and the radiation laws (as well as other information known by the program about system geometry, etc.). If you want to set  $L_2$  independently (perhaps because you have no trust in the radiation laws in a particular situation), set **IPB**=1 and the program will use the input  $L_2$  value. Modes 0 and  $-1$  always accept the input  $L_2$ , so they operate as if **IPB**=1. See Van Hamme & Wilson (1986) for ideas on the use of **IPB** in practical situations.

**IFAT1, IFAT2:** These control whether a blackbody or a stellar atmosphere formulation is used for local emission on stars 1 and 2, respectively. Set **IFAT1**(2)=0 or 1 for blackbody or atmosphere, respectively.

**N1, N2:** These are the grid size integers for stars 1 and 2, respectively. Each is the number of latitude rows per hemisphere. The number of surface elements in longitude scales with **N1**(2) and scales approximately with the sine of the “latitude” coordinate, which runs from 0 at the “North” ( $+z$ ) pole to  $\pi$  radians at the “South” ( $-z$ ) pole.

**VUNIT:** This is the unit for radial velocity input and output, in km/sec. Usually it is a round number, such as 100 km/sec, of the order of the input velocities for **DC**.

**MREF:** The reflection effect can be handled either in detail (viz. Wilson 1990) or by the inverse square law, with corrections for penumbral and ellipsoidal effects. The latter method is much faster and is adequate for many realistic situations. Set **MREF**=1 for the simple treatment and **MREF**=2

for the detailed treatment. It is not advisable to use the detailed treatment for eccentric orbit cases because the required computing time will be almost prohibitively long. Cases for which the detailed treatment is especially recommended include super-synchronous rotators and overcontact binaries.

**NREF**: If detailed reflection is selected (**MREF**=2), then **NREF** specifies the number of reflections in a multiple reflection effect. Set **NREF**=1 for 1 reflection from each star, **NREF**=2 for 2 reflections, etc. More reflections use more computing time. If **MREF**=1, the value of **NREF** is irrelevant.

**IFSMV1**, **IFSMV2**: These integers tell whether spots on stars 1 and 2, respectively, are to move in longitude due to asynchronous rotation and orbital eccentricity. If **IFSMV1** is set to 0, the spots on star 1 remain at fixed longitudes, referenced to the line of centers of the two stars. This behavior is expected for hot spots due to an accretion stream. If **IFSMV1**=1 and the star rotates asynchronously, then the spots on star 1 follow the physical surface as time progresses. This behavior is expected for magnetic spots. In both cases, there is no motion in latitude.

**ICOR1**, **ICOR2**: These integers refer to proximity and eclipse effects on radial velocities for stars 1 and 2, respectively. Value 0 turns the effects off, 1 turns them on.

**IF3B**: This control integer tells the program if there is a third body (**IF3B**=1) or not (**IF3B**=0).

**LD1**, **LD2**: These integers set the limb darkening laws for stars 1 and 2, respectively. **LD1**(2)= $\pm 1$  for the linear cosine law, **LD1**(2)= $\pm 2$  for a logarithmic law, and **LD1**(2)= $\pm 3$  for a square root law. For positive **LD1** or **LD2**, the  $x, y$  coefficients are fixed and equal to the input values. For negative **LD**'s, subroutine **LIMDARK** ignores the input  $[x, y]$  and computes  $x, y$  coefficients locally in terms of  $T_{eff}$ ,  $\log g$  and  $[M/H]$ .

## 9.2. Those for LC only

**HJDST**: The time at which **LC** is to start computing output points. **HJDST** and the next two quantities, **HJDSP** and **HJDIN**, are utilized only if **JDPHS**=1. They are ignored if **JDPHS**=2.

**HJDSP**: The time at which **LC** is to stop computing output points.

**HJDIN**: The time increment for output points. **HJDIN** = 0.001 will produce output points spaced by 0.001 days.

**PHN**: The phase of normalization, which is the phase at which the column of normalized light is normalized to the input value **FACTOR** and the magnitude column is caused to equal the magnitude zero point, whose name is **ZERO**.

**PHSTRT**, **PHSTOP**: The first and last phases at which output points are to be produced. **PHSTOP** should be larger than **PHSTRT**, but neither has to be in the range 0 to 1. For example, **PHSTRT** = -3.2000, **PHSTOP** = 27.4422 is a valid phase range. **PHSTRT**, **PHSTOP**, and the next quantity, **PHIN**, are utilized only if **JDPHS**=2.

**PHIN:** The phase increment for output points.  $\text{PHIN} = 0.020$  will produce output points every 0.020 in phase, within the range **PHSTRT** to **PHSTOP**.

**MPAGE:** This integer is 1, 2, 3, 4 or 5 according to whether the output is a light curve, radial velocity curves, spectral line profile(s), star radii, or sky image coordinates, respectively.

**ZERO:** This is the reference level for output magnitudes (the magnitude at phase **PHN**).

**FACTOR:** This is the scaling factor for the normalized light column. The number in that column will equal **FACTOR** at phase **PHN**.

### 9.3. Those for **DC** only

**DEL's:** Most of the partial derivatives needed by **DC** must be computed numerically. The **DEL's** are the parameter increments applied when approximating those derivatives by finite differences. Some derivatives can be computed analytically and require no **DEL's**. That is why the number of **DEL's** is smaller than the number of parameters. However, the numerical labels of the **DEL's** match those of the parameters (e.g. **DEL** number 25 goes with parameter 25). The order in which the **DEL's** appear in the input lines can be read from the output of **DC**, where they are labeled by parameter name, or from the **DC** input in Appendix B, where they are labeled by parameter number. Give some thought to reasonable values for the **DEL's**. **DEL's** that are too large will cause systematic errors in the derivatives, and **DEL's** that are too small will cause excessive numerical noise.

**KEP** or **KEEP:** These mean the same thing generically. The difference in meaning in a programming sense is not something the user needs to be concerned with. The **KEEP's** determine which parameters are to be adjusted, and have only two possible values, 0 to allow adjustment and 1 to keep a fixed value. There are 60 parameter channels, with numbers 1 to 35 assigned to band-independent parameters and 56 to 60 assigned to band-dependent parameters. Channels 41 to 55 are reserved for future expansion. Accordingly, there are 60 **KEEP's** but only numbers 1 to 35 and 55 to 60 are now meaningful. All **KEEP's** are entered together on an input line, separated into blocks so as to be easy to count off. There is one such input line for the base set, early in the data stream, and a set of  $n$  such input lines at the end of the data stream (for the  $n$  subset solutions). The subsets must indeed be subsets, not supersets, since a given parameter cannot be adjusted in a subset unless it is adjusted in the base set. The order of appearance within the string of 1's and 0's is the same as that of the 60 parameter channels, which are listed in §8 (pp. 19–20) and also printed with every run of **DC**.

**IFDER, IFM, IFR:** Three print control integers follow the **KEEP** integers on the same input lines. Each is set to 1 for “yes, print” or to 0 for “do not print.” **IFDER** decides whether the matrix of derivatives and  $O - C$  residuals, or observational equations (both the unweighted and the weighted matrices), are printed. Usually one would print the derivatives and residuals only for the base set

solution, since the numbers are unchanged in the subset solutions (except for missing columns). IFM decides on the printing of the matrix of normal equations, the matrix of correlation coefficients, the product matrix of the normal equations times their inverse (which should be an identity matrix), checks of the computed dependent variables (obtained by back-substitution of the answers into the normal equations), and the sum of absolute values of residuals (a single number). The user may want to see these numbers for all subset solutions or perhaps only for certain solutions. IFR decides on the printing (after each subset solution) of a block of information about relative radii ( $R_1/a, R_2/a$ ), the derivatives of radii with respect to surface potential and mass ratio, and the standard errors of the radii.

XLAMDA ( $\lambda$ ): The Marquardt multiplier (see p. 28, §10.2). XLAMDA is the final entry on the “KEEP” lines and is the only floating point number on those lines.

KSPA, NSPA, KSPB, NSPB: Each of the two stars may have many spots, but **DC** can adjust the parameters of at most two spots in any one run. This does not imply a limit to the number of spots adjusted overall, since other spots can be adjusted in other runs. Think in terms of spot A and spot B, which may be on the same or different stars. KSPA, NSPA, KSPB, and NSPB are best understood as a set. Given that some spot parameters are to be adjusted (which is determined by KEEP’s 1 to 4 for spot A and KEEP’s 5 to 8 for spot B), the four integers (KSPA, etc.) determine for which of the (perhaps) many spots on the two stars those parameters will be adjusted. KSPA=1 means that spot A is on star 1, and KSPA=2 means that it is on star 2. NSPA determines which spot on the thus determined star is spot A. If KSPA=2 and NSPA=3, then star 2’s third spot is spot A. Star 2 might have some large number of spots, say 10, and spot A will be the third one. Of course, the program will do something crazy if you ask it to adjust the third spot on a star that has only two spots – we have not bothered to find out what. The order assigned to the spots is just the order in which their parameters are read in the input lines. Naturally, KSPB and NSPB apply the same way in identifying spot B. If you want to adjust, say four spots, you can use the MMS (see next section) and adjust two of them in one **DC** iteration, the other two in the next iteration, and so on.

IFVC1, IFVC2: These integers tell **DC** whether or not to expect a radial velocity curve in the input stream for star 1 and star 2, respectively (0 for no, 1 for yes). Velocity curves precede light curves in the input. So with IFVC1=0 and IFVC2=1, **DC** expects the first curve it sees to be a velocity curve of star 2.

NLC: This integer is the number of light curves in the input stream. With IFVC1=1, IFVC2=1, and NLC=3, **DC** expects to encounter two velocity curves (the first for star 1, the second for star 2) and then 3 light curves. It will do a simultaneous adjustment (one iteration) of these five curves, producing one correction for each parameter that is the same for all curves (e.g. mass ratio) and 3 corrections for each parameter that is different for each of the 3 light curves (e.g. limb darkening). The 3 curve-dependent parameter corrections for a given stellar component are printed in direct succession. Input and output parameters, their differences (corrections from input values) and their

estimated standard errors are listed by parameter number (see list in §8, pages 19–20). Curve-dependent parameters are listed also by curve number, according to the input order of the curves. “Curve 0” means “curve-independent.”

**KO:** **DC** can process multiple parameter subsets in a given run, as explained under the method of multiple subsets (MMS, see §10.1 on p. 27). Control integer **KO** provides three options to the user who has established a “scratch pad” data file on the local computer. Such a scratch file should be designated as input-output unit 9. The purpose of this provision is to save computing time by writing all of the numerical derivatives and residuals on the scratch file so that they can be read back for further subset solutions in a later submission (in case you think of some subsets that you first thought were not needed). Weighting can be changed for the re-submission. The later run(s) then will take extremely little computing time, since all the hard computation has been done. This feature is much less important than it once was because of improvements in machine speed and intrinsic program speed, but it is still available. Multiple subsets can be processed within a given run without using the scratch pad (set **KO**=2, **KDISK**=0). **KO** can be 0, 1, or 2, as follows:

**KO**=0. Do the base set and stop. Actually **KO**=0 serves no significant purpose, as simple omission of subsets has the same result.

**KO**=1: Read in the derivatives and residuals that were written to the scratch pad in a previous run. Process these according to the weighting schemes, subsets, and Marquardt  $\lambda$ ’s requested in this run. Do not re-compute any derivatives. A run with this option should take only a few seconds or less of computing time.

**KO**=2: Write the derivatives and residuals generated in this run on the scratch pad for future use. **KO**=2 can be used with **KDISK**=0 if the user does not wish to change the present contents of the scratch pad (or if there is no scratch pad). In this case the program still can process multiple subsets of the parameter list but, of course, cannot come back later to process the data further.

**KDISK:** Set **KDISK**=1 to use the scratch pad, or **KDISK**=0 not to use it. Caution: the program will crash if no scratch pad has been set up and you set **KDISK**=1.

**ISYM:** **DC** can carry out its solutions with either asymmetrical (**ISYM**=0) or symmetrical (**ISYM**=1) derivatives. Solution convergence is somewhat better with symmetrical derivatives, but they take almost twice as long to compute as asymmetrical derivatives. Because of the improved convergence, fewer iterations may be needed with **ISYM**=1, but the individual iterations will take more machine time. Recommendations are to use **ISYM**=0 for strong (well determined) solution situations, and **ISYM**=1 when convergence is a problem or you are in the final fine-tuning stages. Setting **ISYM**=1 is particularly advisable when large increments (**DEL**’s) are used to compute the numerical derivatives. See Wilson & Biermann (1976) or Wilson (1979), Eqns. 20–21.

**NPPL:** (number of points per line) The number of data triplets (time or phase, velocity or light, weight) on each line of the input data stream. The number of points per output line is the same. **NPPL** can be as small as 1 or as large as 5.

**MAGLITE:** Control integer that is 0 if **DC** input is flux (light) or 1 if **DC** input is magnitude.

**N1L, N2L:** These are the coarse (low) grid integers. As explained on p. 34 in §13 (Special Features), they apply to certain derivatives for which computing time can be saved by using coarse grids. Otherwise they are just like N1 and N2.

**NOISE:** This integer is on the same input lines as the curve-dependent parameters for light curves, so there is one value for each light curve (no values for velocity curves). **NOISE** specifies how level-dependent weights are to be applied (see discussion of weighting in §15, p. 36).

**SIGMA:** The estimated standard deviations of the various observed light and velocity curves. **SIGMA** specifies the relative weights of the curves in the solution (weight is inversely proportional to **SIGMA** squared). See discussion of weighting on p. 36 in §15.

**SPHAS1, SPHAS2, SPHAS3, SPHAS4:** The start/stop phases of the two phase ranges for calculating sums of squares of residuals (see §2.2, p. 6).

## 10. Problems with Solution Convergence

There are three main reasons for poor convergence or convergence failure in differential corrections solutions, and it is important to understand all three. The problems can occur in combination, but let us try to understand them separately.

One problem is a broad, shallow minimum in parameter space. This problem can result in slow convergence, even when only a few parameters are being adjusted, if the minimum is very shallow indeed. It is a problem not only for **DC**, but for any minimization scheme, since it is intrinsic to parameter space itself. Nothing can be done except to compute with precision (use a fine grid) and do many iterations.

A second problem, also easy to understand, is lack of precision in computing residuals and derivatives. Since most derivatives are computed numerically in **DC**, we have two obvious sources of error connected with finite differencing, which approximates the true derivative of a synthesized observable,  $f$ , by the slope near to a given operating point, either asymmetrically,

$$\frac{\partial f}{\partial p_n} = \frac{f(p_n + \delta p_n) - f(p_n)}{\delta p_n},$$

or symmetrically,

$$\frac{\partial f}{\partial p_n} = \frac{f(p_n + \frac{1}{2}\delta p_n) - f(p_n - \frac{1}{2}\delta p_n)}{\delta p_n}.$$

Here,  $p_n$  is a parameter and  $\delta p_n$  is its increment. Pseudo-random errors arise from lack of accuracy in the computed  $f$  while systematic errors come from approximation of  $\partial f/\partial p_n$  by  $\delta f/\delta p_n$ . A symmetrical derivative is a better representative of the true derivative, but takes longer to compute. Of course, the systematic error can be reduced by taking smaller  $\delta p_n$ , but the increment should not

be too small or errors in computation of  $f$  will become important. Inaccuracy in computing  $f$  can be reduced by use of a finer numerical grid, but at the cost of increased machine time.

The third problem is more subtle – particularly because it is due to a combination of *two* conditions, neither of which causes convergence failure when acting alone. It is important to realize that this third problem is quite distinct from the other two and can destroy convergence even if the other problems are completely absent. The two interacting conditions that cause the problem are *non-linearity in the appropriate equation of condition* and *parameter correlation*. The first of these conditions can be expressed equivalently by saying that second or even higher order derivatives are required in the differential corrections equation of condition (which normally is written with only first derivatives). One way of seeing into the nature of this problem is to notice that a correct solution algorithm needs to know about parameter correlations, but a linear Least Squares algorithm cannot compute them correctly because it has no knowledge of the higher order derivatives. As a result there will be virtual tradeoffs among the invisible higher order terms that result in incorrect predictions of parameter corrections. It usually is not practical to add second order terms to the equation of condition for several reasons, so two procedures that improve convergence greatly while keeping only linear terms will now be discussed.

### 10.1. Method of Multiple Subsets (MMS)

One realistic remedy for poor convergence is to break the full parameter set into subsets and thus reduce the complexity of the correlations in a given iteration (Wilson & Biermann 1976). This procedure, in which subsets A and B, or A, B and C, etc., are solved iteratively in a closed loop has come to be known as the Method of Multiple Subsets (MMS), and it works very well in practice. It seems that a **DC** solution has only minor difficulty in dealing with one or two fairly strong correlations among parameters. The real difficulty comes when there are many large or even moderately large correlations, and this circumstance accounts for the success of the MMS. Two important points need to be emphasized. First, the published standard errors should come from a final run that includes all adjusted parameters together – not from the subset solutions (which will give unrealistically low error estimates because they see only part of the correlation matrix). It must be said that even the standard errors for the full set will be of questionable accuracy when there is a complicated correlation matrix, but they will have to do in the absence of more rigorously computed standard errors. The second point is that one must resist the temptation to apply the corrections printed in this final run (which is for error estimates only). It is the nature of the problem that, when the MMS is needed, it is needed all the way to the end. Even if you start from the exactly correct Least Squares minimum, the correlation problem discussed here will produce relatively large, erroneous corrections for the full parameter set. This has nothing to do with the accuracy of the derivatives computed by **DC**, but only with the form of parameter space. Some persons have misunderstood this point and assumed that correlation problems occur only away from the correct solution and that they will disappear at the correct solution. Not only do



the problems not then disappear, but they do not even lessen in severity.

## 10.2. Levenberg-Marquardt (L-M)

A very good trick for effecting convergence is the Levenberg-Marquardt procedure (Levenberg 1944; Marquardt 1963), which has been utilized with excellent success in several binary star solution programs such as those by Hill & Rucinski (1993), by Kallrath et al. (1998), and by Djurasevic (1992). The L-M algorithm operates on the matrix of normal equations so as to effect a compromise between the ordinary **DC** corrections (which usually give fast convergence but sometimes lead to convergence failure) and those of the method of steepest descent (convergent but sometimes slow). The normal equations are re-scaled as prescribed by Marquardt, so as to have diagonal elements of unity, although it would be possible to apply the procedure without re-scaling. At the heart of the scheme is a quantity  $\lambda$  that is added to each diagonal element of the re-scaled normal equations. A decision faced by all those who adopt the scheme is that of how to fix the value of  $\lambda$ , and various kinds of iterations have been used in the literature to optimize  $\lambda$ , including one in the original Marquardt (1963) reference. The **DC** program operates on a somewhat different idea, following experience that – at least in binary star problems – a broad range of  $\lambda$ 's typically gives nearly the same corrections. Usually such  $\lambda$ 's are very small, and one finds results for say  $\lambda = 10^{-4}$ ,  $10^{-5}$ , or  $10^{-6}$  that differ by so little that the choice is inconsequential. Because the **DC** program already has been set up to make solutions for many subsets of the main parameter set, it is easy to do solutions for many  $\lambda$  values, so that no iteration on  $\lambda$  is needed. One simply includes a line of data for each such solution. For example, if not sure of the  $\lambda$  value to use, just do solutions (same **DC** submission) for as many as deemed potentially interesting, all for the same parameter set or subset. The lines will look like this:

```
1111 1111 0111100 01110 11010 00110 01000 11111 11111 11111 11111 01110 0 1 0 1.000d-06
1111 1111 0111100 01110 11010 00110 01000 11111 11111 11111 11111 01110 0 1 0 1.000d-05
1111 1111 0111100 01110 11010 00110 01000 11111 11111 11111 11111 01110 0 1 0 1.000d-04
```

where the integer 1's and 0's are the KEEP's and print control integers for a subset solution, while the final 1.000d- $n$  is  $\lambda$  for that solution. The execution time to do the several solutions (even very many) is negligible. Usually it makes little difference which  $\lambda$  value is adopted within such a range, although  $10^{-11}$  or  $10^0$  may give much different answers.

## 10.3. MMS or L-M?

Either the MMS or Levenberg-Marquardt algorithm will nicely solve most convergence problems of the correlation – non-linearity type. The L-M algorithm is the more convenient because it does not extend the level of iteration. However, L-M alone cannot handle the most severe convergence problems, where a combination of L-M and MMS may be needed.

#### 10.4. Error Distributions and Standard Error Estimates

Estimated standard errors are printed along with the parameter corrections for the base set and subset solutions. Published comments occasionally appear to the effect that the **DC** program produces unrealistic error estimates, although the errors are computed from the covariance matrix by the standard method. For example, a 1997 paper comments about **DC**: “... *the estimates of errors of the adjustable parameters are unrealistically small. The reason is partly the strong correlation between the relatively many parameters, and partly the non-normal distribution of measurement errors.*” Such remarks have some formal validity, but are misleading for the following reasons:

1. *Strong correlation between the relatively many parameters*: In significantly non-linear situations the standard errors will be only approximately right, but that is also the case with all other binary star fitting programs that now exist. The shortcoming is not peculiar to **DC**. However, an impression that the problem *is* peculiar to **DC** obviously has arisen, and one can speculate as to why that is so. An obvious point is that some solution programs avoid such complaints by not computing error estimates at all, or by doing so with invented rules that produce comfortably large estimates. However, some other solution programs do compute standard error estimates, *but usually not for adjusted mass ratios*. Experience shows that mass ratios cause most of the problems with fitting and with error computation. This is so because the mass ratio often has major correlations with other parameters, and it also shows particularly non-linear behavior. Programs that cannot adjust the mass ratio parameter (most programs) will not encounter the problem.
2. *The non-normal distribution of measurement errors*: The cited phrase probably does not literally refer to *measurement errors*, whose distributions for light curves are as close to Gaussian as one is likely to find for any astrophysical measurements. (Observers measure light with respect to nearby comparison stars whose brightness and color are close to that of the variable star. The observers typically expend considerable thought and effort to eradicate possible systematic errors.) The phrase more likely refers to *non-normal distribution of residuals*, which one finds when a real binary does not match the model very well (perhaps it has un-recognized spots or circumstellar gas, etc.), or if the solution is at an incorrect local minimum. If interpreted in this way, the statement is quite correct, although trivial. Under any interpretation, the comment has nothing to say about **DC**.

D. Terrell has carried out experiments by solving synthetic light curves by means of **DC**. The light curves contained simulated Gaussian errors and, of course, their parameters were known. He found that the parameter errors (solution-known) agreed with those expected from the standard errors in output from **DC**.

## 11. Interactive Branching and the Lack of Automatic Iteration in DC

Many persons have been surprised at having to re-submit the program at each **DC** iteration. The lack of automatic iteration is not an oversight, but a way to cause users to look at the progress of a solution. Even without automatic iteration, some persons believe whatever comes out of the machine, even when the results violate common sense. In this highly non-linear problem, the solution process can indeed get into trouble and produce inappropriate corrections. To some extent it is possible to have a program detect inconsistent and strange results. **DC** has a few such kinds of detection and message generation, and further messages about such problems may be worked into future versions. However, until such screening has been made very extensive, the user is encouraged to think about every iteration.

There is another important reason to avoid automatic iteration. In real situations, an experienced user will decide which parameters to adjust and which to hold fixed as part of a “dynamical” process of interaction with the iterative solution. Occasionally the set of adjusted parameters will remain unchanged from start to finish, but usually it will not. Because of this reality, **DC** is set up for interactive branching, by which each iteration produces solutions of as many subsets of the main (base) set as are requested (control integer **KO** must be set to 2). Notice that the sample input data supplied with the program requests several subset solutions via extra lines of **KEEP**’s at the end of the data block. There is no limit to the number of such subset solutions – you can ask for a hundred or more if you like – and they require almost no machine time since solving the observational equations takes almost no time compared to generating them. The only penalty for requesting a large number of subset solutions is the proliferation of output, with possible attendant confusion. So the idea is to decide, after each iteration, which set of adjustable parameters to follow from that point on (to the next such change). Interactive branching is discussed in Wilson (1988).

However, the most cogent reason for personal monitoring of **DC** iterations is simply this: *There is more to astrophysics than parameter estimation*. Recognition of strange and unusual behavior points the way to the really major insights and discoveries. Many close binaries are not well represented by even our best generic models, and in some cases the problem is not due just to approximate computation but to quite unsuspected features. To drive this point home, imagine applying one’s favorite “standard” light curve model automatically to one of the classic strange objects – perhaps one with an unusual kind of circumstellar disk. Persons familiar with  $\beta$  Lyrae might be amused by imagining the outcome for that binary. One will get answers (meaningless ones, naturally), but will have at best only a much reduced chance for insights into the real problems. Often one really needs to get in there and watch things develop.

## 12. Radiative Physics

The radiative treatment was re-done for 2003, with the previous effective wavelength prescription replaced by a bandpass prescription (Van Hamme & Wilson 2003), based on model stellar atmo-

spheres by Kurucz (1993). The radiative routine computes bandpass-integrated normal emergent intensities, as well as Planck intensities, by means of Legendre polynomials whose coefficients are in data files `atmcof.dat` and `atmcofplanck.dat`. A path to these files must be set within the computing system or they must be in the same sub-directory as the programs. Besides 25 bands (viz. Table 2, p. 33), there are coefficients for 19 chemical compositions and 11 surface gravities (Table 1, p. 32). Note that if local interpolation in  $(T_{eff}, \log g)$  for  $x$  and  $y$  limb darkening coefficients is requested (with the law-specifying integers LD1 and/or LD2 being negative, see §9.1, p. 22), a path to the bandpass-specific limb darkening tables of Van Hamme (1993) must be set or the limb darkening files must reside in the same sub-directory as the program. The limb darkening tables are in data files named `limcof_bp_*.dat`, with the asterisk to be replaced by the chemical composition indicator, and can be downloaded from the WWW site <http://www.fiu.edu/~vanhamme/limdark.htm>.

### 12.1. Radiative Parameters and Control Integers

Temperature ranges vary according to  $\log g$  and are listed in Table 1 on p. 32, together with 19 abundances (relative to the Sun). Bands, band identification numbers (IBAND), and references for band response curves are in Table 2 on p. 33. Chemical composition needs to be specified via input parameter ABUNIN. If ABUNIN is not one of the 19 Kurucz values (Table 1, p. 32), it is reset automatically to the nearest Kurucz value, as there is no interpolation in  $[M/H]$ . (**Important note:** program logic requires that data files `atmcof.dat` and `atmcofplanck.dat` remain unchanged. Please make these files “write protected” so they cannot be changed inadvertently.)

### 12.2. Atmosphere to Blackbody Transitions

If a  $[T_{eff}, \log g]$  combination is outside the range of applicability (see Table 1 on p. 32), the program smoothly transfers to the Planck intensity curve for the given band over a predetermined range that is coded into the program. We call the procedure ramping. Four ramp parameters are required, two for  $\log g$  that set the ramp interval below  $\log g = 0.0$  (GLOWTOL) and above  $\log g = 5.0$  (GHIGHTOL), and two for  $T_{eff}$ . At the lower  $T_{eff}$  limit, there is only one temperature ramp interval, TLOWTOL. For each  $\log g$ , the ramp interval at the upper  $T_{eff}$  limit is a fixed percentage of the maximum ‘atmosphere’ temperature. This fixed percentage is determined from the ramp interval of the highest model temperature of 50,000 K (THIGHTOL). The program, as supplied, values GLOWTOL = 4.0, GHIGHTOL = 4.0, TLOWTOL = 1500 K, and THIGHTOL = 50,000 K. As an example of what this means, consider the THIGHTOL of 50,000 K coupled with a highest Kurucz  $T_{eff}$  of 50,000 K. The **LC** and **DC** programs will then apply Kurucz atmosphere intensities for local surface elements with  $T_{eff}$  up to 50,000 K, smoothly transfer between the 50,000 K atmosphere and a 100,000 K blackbody between 50,000 K and 100,000 K, and use blackbody intensities above 100,000 K. GLOWTOL and GHIGHTOL were smaller prior to February, 2004. The current numbers seem to work well but the user may change them. They should be the same in **LC** and **DC**.

Table 1. Temperature Limits

[M/H] <sup>a</sup>	$\log g$ (cgs)	$T_{eff}$ Range (K)	[M/H] <sup>a</sup>	$\log g$ (cgs)	$T_{eff}$ Range (K)
−5.0 to 0.5	0.0	3500 to 6000	−5.0	3.5	4000 to 31000
1.0	0.0	3500 to 5750	−4.5 to −4.0	3.5	3750 to 31000
−5.0 to −3.5	0.5	3500 to 7000	−3.5 to −2.5	3.5	3500 to 31000
−3.0	0.5	3500 to 7250	−2.0	3.5	3500 to 29000
−2.5 to 0.2	0.5	3500 to 7500	−1.5 to 0.1	3.5	3500 to 31000
0.3	0.5	3500 to 7250	0.2 to 0.3	3.5	3500 to 30000
0.5	0.5	3500 to 7000	0.5	3.5	3500 to 29000
1.0	0.5	3500 to 7250	1.0	3.5	3500 to 27000
−5.0 to −3.0	1.0	3500 to 7500	−5.0 to −4.5	4.0	4250 to 35000
−2.5	1.0	3500 to 7750	−4.0	4.0	4000 to 35000
−2.0 to −1.5	1.0	3500 to 8000	−3.5	4.0	3750 to 35000
−1.0	1.0	3500 to 8250	−3.0 to −2.5	4.0	3500 to 35000
−0.5 to −0.1	1.0	3500 to 8500	−2.0	4.0	3500 to 32000
0.0	1.0	3500 to 8250	−1.5 to −0.1	4.0	3500 to 35000
0.1 to 0.3	1.0	3500 to 8500	0.0	4.0	3500 to 39000
0.5 to 1.0	1.0	3500 to 8250	0.1 to 0.2	4.0	3500 to 35000
−5.0 to −3.0	1.5	3500 to 9000	0.3	4.0	3500 to 34000
−2.5 to −1.5	1.5	3500 to 8500	0.5	4.0	3500 to 33000
−1.0 to 0.5	1.5	3500 to 9000	1.0	4.0	3500 to 31000
1.0	1.5	3500 to 8750	−5.0	4.5	4750 to 40000
−5.0 to 0.1	2.0	3500 to 14000	−4.5	4.5	4250 to 40000
0.2 to 0.5	2.0	3500 to 10500	−4.0	4.5	4500 to 40000
1.0	2.0	3500 to 10000	−3.5 to −3.0	4.5	4000 to 40000
−5.0 to 0.2	2.5	3500 to 19000	−2.5	4.5	3750 to 40000
0.3	2.5	3500 to 18000	−2.0 to −0.1	4.5	3500 to 40000
0.5	2.5	3500 to 17000	0.0	4.5	3500 to 49000
1.0	2.5	3500 to 11000	0.1 to 0.3	4.5	3500 to 40000
−5.0	3.0	3500 to 26000	0.5	4.5	3500 to 37500
−4.5	3.0	3750 to 26000	1.0	4.5	3500 to 35000
−4.0 to −3.5	3.0	3500 to 26000	−5.0	5.0	5000 to 50000
−3.0	3.0	3500 to 27000	−4.5 to −3.5	5.0	4500 to 50000
−2.5 to 0.1	3.0	3500 to 26000	−3.0 to −2.5	5.0	4250 to 50000
0.2 to 0.3	3.0	3500 to 25000	−2.0 to 0.3	5.0	3500 to 50000
0.5	3.0	3500 to 24000	0.5	5.0	3500 to 45000
1.0	3.0	3500 to 21000	1.0	5.0	3500 to 40000

<sup>a</sup> $\log(\frac{M}{H}) - \log(\frac{M}{H})_{\odot}$

Table 2. Bandpass List

Bandpass	IBAND	Reference	Bandpass	IBAND	Reference
<i>u</i>	1	1	<i>R<sub>C</sub></i>	15	5
<i>v</i>	2	1	<i>I<sub>C</sub></i>	16	5
<i>b</i>	3	1	230	17	6
<i>y</i>	4	1	250	18	6
<i>U</i>	5	2	270	19	6
<i>B</i>	6	3	290	20	6
<i>V</i>	7	3	310	21	6
<i>R</i>	8	4	330	22	6
<i>I</i>	9	4	TyB	23	7
<i>J</i>	10	4	TyV	24	7
<i>K</i>	11	4	HIP	25	7
<i>L</i>	12	4			
<i>M</i>	13	4			
<i>N</i>	14	4			

Note. — Response curves for bands 17 to 22 are rectangular, have widths of 20 nm and are centered on the wavelength (in nm) indicated by their names. They are useful for certain space-based observations in the UV.

References. — (1) Crawford & Barnes (1974); (2) Buser (1978); (3) Ažusienis & Straižys (1969); (4) Johnson (1965); (5) Bessell (1983); (6) Kallrath et al. (1998); (7) Bessell (2000).

### 13. Special Features

**Coarse and fine grids:** In order to save computing time, **DC** uses both coarse and fine computing grids, and applies the fine grids only where accuracy requires them. The fine grids are for the residuals,  $[(O - C)]$ 's, and for derivatives with respect to parameters  $e, \omega, F_1, F_2, \phi_0, i, \Omega_1, \Omega_2, q$ , spot latitude, spot longitude, and spot radius. Fine grids are required for parameters that are in some way geometrical. The derivatives of the non-geometrical parameters  $g_1, g_2, T_1, T_2, A_1, A_2, L_1, L_2, x_1, x_2$ , and spot temperature are computed with the coarse grids. The fine grids are specified by **N1, N2** and the coarse grids by **N1L, N2L**, for stars 1 and 2, respectively.

**Synthetic Noise:** Light curves with synthetic Gaussian noise can be produced by entering a non-zero value for the input quantity **STDEV**, which is labeled “fract. sd.” in the output. As the label suggests, **STDEV** is in the unit of the light at the reference phase (**PHN**). The way that the noise (scatter) scales with light level is controlled by input integer **NOISE**. Scatter is proportional to light level for **NOISE=2**, proportional to the square root of the light level for **NOISE=1**, and independent of light level for **NOISE=0**. The random number generator needs a seed (FORTRAN name **SEED**), labeled “seed” in the output. **SEED** should be larger than 100000001. and smaller than twice that value. The procedure should work for other **SEED**'s, but one should stay within the mentioned range for best results. Noise generation works the same way for radial velocities, except that there is no variation of the scatter with level. For velocity scatter, enter **STDEV** in the velocity unit (**VUNIT**). Users who have a favorite random number generator should find it easy to replace the program generator with their own.

### 14. Simultaneous Solutions

A capability of **DC** that should be used in most circumstances is that of making simultaneous multi-band light curve solutions as well as simultaneous (one or two curve) radial velocity and multi-band light curve solutions (Wilson 1979). The advantages are to avoid inconsistencies among solutions of the separate curves, to reduce the number of free parameters (no need to have both photometric and spectroscopic mass ratios, nor to have a separate inclination for each light curve), and to utilize information that is discarded in separate solutions (the knowledge that there is only one true mass ratio, one true eccentricity, etc.). Weighting is very important in simultaneous solutions and is discussed in Wilson (1979) and on p. 36 in §15. The sample **DC** data supplied with the program are for a simultaneous solution of two radial velocity and three light curves. There can be 0, 1, or 2 velocity curves and any number of light curves (the program is dimensioned for up to 50 light curves, which seems more than ample for realistic situations). Input integers **IFVC1** and **IFVC2** tell the program how many velocity curves to expect, and **NLC** tells it how many light curves to expect. The **DC** solution will produce one correction for each adjusted curve-independent parameter (most of them) and  $n$  corrections for each adjusted curve-dependent parameter ( $L_1, L_2, x_1, x_2, \ell_3$ ). It takes no more (actually somewhat less) machine time for a simultaneous solution of

$n$  curves than for  $n$  individual curve solutions, so there really is no reason not to take advantage of this feature. If one has misgivings about the simultaneous solution, separate solutions always can be carried out in addition. One particularly illogical practice in a few papers has been to publish averaged parameters from separate solutions and to offer the averages as a substitute for a simultaneous solution. However, the average of separate solutions will not be a correct solution of *any* of the separate curves. Ask “what is the proper way to take such an average?” The answer is that there is no self-consistent way – the only way to have the results of a simultaneous solution is to *do* a simultaneous solution.

## 15. Input Lines, Including Observations and Weights

Identification of the various input quantities can be done in several ways, for example via Appendices A or B, by examining the output where the numbers are printed with labels, or by comparing the input data lines with the FORTRAN READ statements.

For **LC** (see Appendix A, p. 48), there is a “more or less standard” set of input lines with control integers and parameters for each curve, and  $n$  such sets of lines can be concatenated to produce  $n$  output curves. Output can be a light curve, a radial velocity curve, or a spectral region containing mixed absorption and emission spectral lines (perhaps blended). The “standard set” of input data lines is only “more or less standard” because there can be extra lines to tell about circumstellar attenuating clouds and/or about bright or dark star spots and/or about spectral lines. For light or velocity curves, the number of input lines for a given curve is  $(11 + c + s)$ , where  $c$  is the number of circumstellar clouds and  $s$  is the number of star spots for the two stars combined. One of the 11 mandatory lines is the *stop* line for clouds and two others are stop lines for star spots, one to follow the lines of spot parameters for each star. The cloud stop line should contain a number greater than 100. but less than 200. in the first field, which corresponds to a cloud  $x$ -coordinate. Each of the two stop lines for spots should contain a number greater than 200. in the first field, which corresponds to a spot latitude on the normal spot parameter lines. According to the value of **MPAGE**, **LC** computes a light curve (**MPAGE**=1), a radial velocity curve (**MPAGE**=2), a set of spectral regions (**MPAGE**=3), star dimensions (**MPAGE**=4), or plane of sky coordinates for images (**MPAGE**=5). Spectral line profile computation requires extra input lines for the line characteristics. Therefore, the sample data include an input file for line profiles (called `lcin.dat3`). The sample files for light curves and velocity curves are called `lcin.dat1` and `lcin.dat2`, respectively. For dimensions and images they are `lcin.dat4` and `lcin.dat5`, respectively. The several sets of input lines need not be of the same binary, since each curve computation is an independent operation. A final line stops execution and should contain integer 9 in the field normally occupied by **MPAGE** (that final line is not counted in the  $11 + c + s$  lines of the individual curves).

For **DC** (see Appendix B, p. 50), three lines of **DEL**’s enter first. The next line pertains to the base parameter set and contains 60 **KEEP**’s (20 of which are not used at present) for the 40 adjustable parameters, 3 print control integers (**IFDER**, **IFM**, and **IFR**), and the Marquardt  $\lambda$  (**XLAMDA**).



Next comes a line of spot identification integers (KSPA, NSPA, KSPB, NSPB), then six lines of control integers and curve-independent parameters, then a line of third body parameters, then as many lines of curve-dependent parameters as there are observed curves, then as many lines of spot parameters as there are spots (and also two spot “stop” lines), then as many lines of circumstellar cloud parameters as there are clouds (and a cloud “stop” line), then the observed radial velocities, then the observed light curves, and finally as many lines of KEEP’s for subset solutions as the user wants. These final KEEP lines are of the same format as the KEEP line for the base set and also contain the 3 print control integers and the Marquardt  $\lambda$ . Note that the solutions that are triggered by these KEEP, etc., lines can be for varied  $\lambda$  as well as for parameter subsets. For example, one can run two solutions for exactly the same parameters but with different  $\lambda$ .

The observed radial velocity curves and light curves of the **DC** input data stream are entered as triplets (time or phase, velocity or light, weight), with up to 5 triplets (i.e. data points) per line. The number of data points per line is set by input integer NPPL (*number of points per line*). A possible formatting problem, caused by velocities not being of the same order of magnitude as light measures, is solved by providing for the separate entry of a convenient sized velocity unit. The (0, 1, or 2) velocity curves and NLC light curves are separated by “data stop lines” that serve two purposes. They identify the last data line for each velocity or light curve and also tell how many data points are on that last line (1, 2, 3, 4, or 5). The only number on a data stop line should be in the first field and should be  $-(10000 + k)$ , where  $k$  is the number of data points on the preceding line. Example: if there are 2 data points on the preceding line, the “stop” number should be  $-10002$ . Some computing systems require “something” in the remaining fields of the line and blanks are suitable, but there must be at least blanks (not an absence of all characters). No special stop information is to follow the final curve because the program already knows how many curves to expect, and it knows which is a velocity curve and which is a light curve because it has already read IFVC1, IFVC2, and NLC. After the data stop line are the “KEEP” lines for subsets of the main (i.e. base) set, with one KEEP line for each subset. A line with integer 2 in column 2 should follow the final line of subset KEEP’s to signal the end of subset processing and of the entire job.

Weighting of observations is discussed in Wilson (1979, pp. 1064-1065), Wilson (1988) and Kallrath & Milone (1999). Briefly, the program applies three kinds of weights, which are “intrinsic” weights (assigned by the user to the individual observations), “curve-dependent” weights (based on estimated standard deviations [SIGMA’s] at a reference phase), and “level-dependent” weights (computed by **DC** according to the input parameter **NOISE**, which tells how observational scatter scales with light level). **NOISE** should be set to 1 for scatter that scales with the square root of the light level, such as counting statistics, and to 2 for scatter that scales with the light level, such as scintillation noise or fluctuations in sky transparency. If **NOISE** is set to 0, no level-dependent weighting is applied. Level-dependent weighting does not apply to velocity curves. **SIGMA**’s pertain to the directly entered velocity and light values. An exception is where the **SIGMA**’s were actually measured from individual observations, but normal points (i.e. averages) are entered. In that case, the number of individual points in each normal point should be incorporated in the “intrinsic”

weight of each point. Some persons have contrived their curve-dependent weighting (through the SIGMA's) so as to increase the influence of one or a subset of the curves according to pre-conceptions – for example, of the relative importance or radial velocity and light curves. However, the SIGMA's should properly be based on measurements, not prejudice.

## 16. The Scaling of Run Time

The main computational activity of **LC** consists of summations over surface grid elements in forming the observable fluxes and velocities. The number of grid elements on each star is essentially proportional to the square of the grid fineness integer for that star ( $N_1, N_2$ , number of latitude rows per hemisphere), so run time very nearly scales with  $N_1^2 + N_2^2$ . Computations of all other kinds take negligible time. Of course, run time in **LC** also scales with the number of phase points, or more precisely with that number plus 1 (because one extra point is done at the phase of normalization). Eccentric orbits take longer than circular orbits because the local physical computations must be done anew at each phase. Note that a very small eccentricity (say 0.000001) differs in this regard from one of exactly zero because the program skips the extra computations only if  $e = 0$ . The actual time factor depends on whether the old approximate (MREF=1) or detailed (MREF=2) reflection model is specified. The MREF=2 case takes longer for both circular and eccentric orbits, but it takes *much* longer (usually impractically longer) for eccentric orbits. If MREF=2, then run time also depends on NREF, the number of multiple reflections. There will also be some dependence on the particular binary star configuration. For example, runs for stars with small  $r = R/a$  go faster than those with large  $r$ . The situation is sufficiently complicated so that actual speed factors are best estimated via a few experiments at low grid fineness.

For **DC**, many of the same considerations apply as for **LC**. However, there are four grid fineness integers because **DC** uses both a high and a low grid for each star (viz. §13, p. 34), so run time scales with  $(P + 1)(N_1^2 + N_2^2) + P_L(N_1^2 + N_2^2)_L$ , where  $P$  is the number of fine grid parameters and  $P_L$  is the number of low (coarse) grid parameters under adjustment in the main (base) set of parameters. The first term involves  $P + 1$  rather than just  $P$  because not only derivatives but also residuals,  $(O - C)$ 's, must be computed, and the residuals are done with the fine grid. Run time in **DC** also scales with the number of observations (not with the number of observations plus 1 because there is no phase of normalization, as there is with **LC**). The numbers of parameters in subset solutions are not relevant to run time because run time for the subsets is negligible. **DC** iterations with ISYM=1 (i.e. symmetrical derivatives) will take nearly twice as long as those with ISYM=0 (i.e. asymmetrical derivatives).

## 17. Common Difficulties

1. **Minimum dimensioning:** One of the most frequent reasons for failure in program execution is under-dimensioning of arrays. These failures are disconcerting because very strange things can happen and usually no logical interpretation of the machine error messages is apparent. Array dimensions need be changed only in the main programs (**LC** and **DC**), not in the subroutines. This can be done in the “Parameter Wrapper” at the beginning of the main programs (§21, p. 44). See Table 3 (Dimensioning vs. Grid Fineness) on p. 39 for minimum dimensions of the arrays RV, GRX, GRY, GRZ, RVQ, GRXQ, GRYQ, GRZQ, SLUMP1, SLUMP2, SRV, SGRX, SGRY, SGRZ, SRVQ, SGRXQ, SGRYQ, SGRZQ, SRVL, SGRXL, SGRYL, SGRZL, SRVQL, SGRXQL, SGRYQL, SGRZQL, SLMP1, SLMP2, SLMP1L, SLMP2L, FR1, FR2, GLUMP1, GLUMP2, GRV1, GRV2, XX1, XX2, YY1, YY2, ZZ1, ZZ2, GMAG1, GMAG2, CSBT1, CSBT2, RF1, RF2, RFTMP, SXX1, SXX2, SYY1, SYY2, SZZ1, SZZ2, SGMG1, SGMG2, SGRV1, SGRV2, SGLM1, SGLM2, SCSB1, SCSB2, SRF1, SRF2, SGLM1L, SGLM2L, SGRV1L, SGRV2L, SXX1L, SXX2L, SYY1L, SYY2L, SZZ1L, SZZ2L, SGMG1L, SGMG2L, SCSB1L, SCSB2L, SRF1L, SRF2L, ERV, EGRX, EGRY, EGRZ, ELMP1, EGLM1, EGRV1, EXX1, EYY1, EZZ1, EGMG1, ECSB1, ERF1, ERVQ, EGRXQ, EGRYQ, EGRZQ, ELMP2, EGLM2, EGRV2, EXX2, EYY2, EZZ2, EGMG2, ECSB2, ERF2, ERVL, EGRXL, EGRYL, EGRZL, ELMP1L, EGLM1L, EGRV1L, EXX1L, EYY1L, EZZ1L, EGMG1L, ECSB1L, ERF1L, ERVQL, EGRXQL, EGRYQL, EGRZQL, ELMP2L, EGLM2L, EGRV2L, EXX2L, EYY2L, EZZ2L, EGMG2L, ECSB2L, ERF2L, SFR1, SFR!L, ERF1, ERF1L, SFR2, SFR2L, EFR2, and EFR2L. The minimum dimension of each of these arrays depends on the grid fineness. For example, if the grid fineness integer ( $N1$ ,  $N2$ ,  $N1L$ , or  $N2L$ ) is 30, the minimum dimension is 762. The arrays **STLDH**, **STLDL**, **ETLDH**, and **ETLDL** are dimensioned to the sum of the minimum dimensions for both stars (in above long list of arrays). For  $N$ ’s of 30, this would then be  $762 + 762 = 1524$ . The arrays **PHAS**, **FLUX**, and **WT** are dimensioned to include all the observations in all bands *plus* the blanks on the last data lines *plus* the blank observations on the data stop lines (see **DC** sample input file). Arrays **OBS** and **HOLD** are dimensioned to [(number of observations)  $\times$  (number of parameters in Least Squares solution +1)]. A curve-dependent parameter counts  $n$  times for  $n$  light curves. Thus if you had 98 observations in star 1’s velocity curve, 102 in star 2’s velocity curve, 470 in one light curve, and 530 in another, the first factor would be  $1200 = 98 + 102 + 470 + 530$ . Then, if you adjust  $i$ ,  $g_2$ ,  $L_2$ , and  $x_1$ , the second factor is  $7 = 2 + 2 \times 2 + 1$  (remember,  $L_2$  and  $x_1$  are curve-dependent and count twice each, since there are two light curves. The last +1 is for the residuals. **OBS** and **HOLD** need then be dimensioned to a minimum of  $1200 \times 7 = 8400$ .

2. The parameter increments must be neither too large (gives systematic errors) nor too small (gives numerical noise). These increments are called the DEL’s (FORTRAN name). For most parameters, DEL’s of about 1% of the parameter value are appropriate. However, particular circumstances sometimes affect that guideline, so common sense and experience are the best guides. Finer grids allow smaller DEL’s. Use of the **ISYM=1** option allows larger DEL’s before curvature effects become important.

3. It is best to have the initial parameter guesses for differential corrections based on experiments

Table 3. Dimensioning vs. Grid Fineness

$N$	Minimum Dimension <sup>a</sup>	$N$	Minimum Dimension <sup>a</sup>	$N$	Minimum Dimension <sup>a</sup>
1	...	21	378	41	1418
2	4	22	413	42	1482
3	9	23	452	43	1555
4	16	24	491	44	1628
5	23	25	530	45	1703
6	33	26	575	46	1777
7	45	27	617	47	1857
8	58	28	667	48	1935
9	72	29	713	49	2013
10	88	30	762	50	2098
11	106	31	813	51	2180
12	125	32	867	52	2269
13	147	33	919	53	2352
14	169	34	978	54	2445
15	195	35	1034	55	2533
16	220	36	1092	56	2628
17	250	37	1157	57	2722
18	278	38	1217	58	2818
19	310	39	1281	59	2915
20	342	40	1345	60	3011

<sup>a</sup>For arrays mentioned on p. 38

with the light-velocity program (**LC**). Column 5 in the main block of output from **LC**, which is  $\ell_1 + \ell_2 + \ell_3$ , should approximately match the observed light values.

4. The distinction between direct light (column 5 of **LC** output) and normalized light (column 6) can be a source of confusion. Remember that normalized light is only intended for convenience in initial graphical trials and that it has no counterpart in **DC**, which deals with direct light only. If this continues to confuse you, forget that normalized light exists and work always with direct light.

5. Always check  $\Omega_1$  and  $\Omega_2$  to be sure they are in the permitted range for given  $q$ ,  $F$ , and  $e$ . Exact lobe filling or otherwise special  $\Omega$ 's that are computed by the program in modes -1, 4, 5, and 6 will be correct and need not be checked. In differential corrections, be sure that both the input and incremented values of  $\Omega_1$  and  $\Omega_2$  are within allowed ranges. Overcontact  $\Omega$ 's should be between the critical values for inner and outer contact (see table `critout.tab` of H critical  $\Omega$ 's). For detached stars,  $\Omega$  is greater than the critical  $\Omega$  for inner contact.

6. Certain parameters cannot be adjusted in certain program modes. The reason is either that the parameters are not free, but functionally determined from the mode logic, or that they have no effect on the computed light or velocity. Attempts to adjust those parameters are the most common cause of blowups in subroutines **SQUARE** and **DMINV**.  $L_2$  cannot be adjusted in any mode greater than 0 unless **IPB** has been set to 1. In mode -1, parameters  $g_1$ ,  $T_1$ ,  $T_2$ ,  $A_1$ ,  $\Omega_1$ ,  $\Omega_2$ , and  $x_1$  cannot be adjusted. In mode 1, parameters  $g_2$ ,  $T_2$ ,  $A_2$ ,  $\Omega_2$ ,  $L_2$ , and  $x_2$  cannot be adjusted. In mode 3,  $\Omega_2$  and  $L_2$  cannot be adjusted. In mode 4,  $\Omega_1$  and  $L_2$  cannot be adjusted. In mode 5,  $\Omega_2$  and  $L_2$  cannot be adjusted. In mode 6,  $\Omega_1$ ,  $\Omega_2$ , and  $L_2$  cannot be adjusted. Limb darkening coefficient  $x_2$  cannot be adjusted in any mode if **LD1** or **LD2** is negative (i.e. if limb darkening coefficients are computed as functions of local temperature and other quantities).

7. Luminosities (input  $L_1$  and  $L_2$ ) are approximately  $4\pi$  times larger than computed light values (output  $\ell$ 's). Therefore, to obtain  $\ell_1 + \ell_2$  of about unity outside eclipse, enter input luminosities that add to about  $4\pi$ . Note that the traditional treatment of third light as if it were third luminosity is incorrect (for a discussion of luminosity vs. light, see §4, p. 9).

8. Critical  $\Omega$ 's depend on  $e$ . Although a table of lobe filling circular orbit potentials for synchronous and non-synchronous rotation is available along with this document (file `critout.tab`), such tables for eccentricity and rotation combined would be too extensive for practicality. Therefore, to find critical eccentric  $\Omega_1$  and  $\Omega_2$ , run **LC** in mode 6. The program will replace your input  $\Omega$ 's for both stars with the critical  $\Omega$ 's in the output listing.

9. Overcontact binaries, such as W UMa stars, require finer grids than do detached and semi-detached binaries because the numerics of the neck region are particularly difficult to treat accurately.  $N$ 's about 50% larger than normal are recommended for overcontact systems.

10. The programs can apply the detailed reflection model of Wilson (1990) for eccentric as well as circular orbits, but eccentric cases then use an enormous amount of machine time. If you are doing an eccentric binary and the programs run almost forever, check to see if you set **MREF**=2.

In most realistic situations, the old approximate reflection (MREF=1) should be entirely adequate for eccentric binaries.

11. A very common cause of failed runs is simply inadvertent shifting of numbers on the input lines out of their proper fields, so that leading digits or signs are clipped away, or the numbers are not read at all, or they are read under the wrong name. Be sure to keep a copy of the sample data in exactly its original form for later comparison.

12. Sometimes users make changes that cause the programs not to work. Perhaps the program will work for the immediate application, but will fail in another situation. Be sure to keep a copy of the entire program in exactly the form supplied, so that you can see whether it runs correctly in the circumstances under question.

13. Be sure not to mix subroutines from program versions.

## 18. Summary: Differences from Pre-1992 Versions

1. Star spot parameters can now be adjusted.

2. One can now use an optional non-linear limb darkening law (pre-1992 versions had only the linear cosine law).

3. The reflection effect now can be computed either with the detailed model of Wilson (1990) or with the approximate reflection model of the old program (which is faster). Multiple reflection is included in the detailed model.

4. The present **LC** and **DC** programs are much faster than pre-1992 versions.

5. The specification of phase range in **LC** was made more convenient than before, with allowance for phases outside the range 0 to 1.

6. The stars now orbit counter-clockwise in the sky (for  $i < 90^\circ$ ), rather than clockwise as in the old program. This makes a difference only for pictures of the binary (MPAGE=5).

7. Messages about exceeding limiting lobes are now generated if the stars exceed the lobes at all, instead of only when at least one grid point falls in the hole near the inner Lagrangian point, as in the old program.

8. Star spots now can optionally move in longitude, keeping pace with the physical surface of an asynchronously rotating star, rather than being tied to the coordinate grid.

9. For both stars, **LC** now provides absolute mass, bolometric luminosity, equivalent sphere radius, and approximate absolute mean surface gravity.

## 19. Summary: Differences Between 1998 and 1992 Versions

1. The output from **LC** now is determined by setting integer MPAGE to 0, 1, 2, 3, 4, or 5. Older versions had light and velocity curve output on the same pages, which made the page format inconveniently wide. The output should now be easier to read. The input file format now differs for the various values of MPAGE (samples are provided with the distributed program).
2. **DC** solutions can include the Marquardt  $\lambda$  factor.
3. The Least Squares normal equations are in re-scaled form (diagonal terms are unity before application of  $\lambda$ ).
4. Semi-transparent circumstellar clouds (at fixed locations in the rotating frame) can be included.
5. Rotational spectral line profiles can be computed (other broadening mechanisms no; blending yes).
6. The program now is entirely in double precision.
7. Non-linear limb darkening via a square root law is an added option (1992 version had only the logarithmic and linear laws).
8. One can now use either time or phase as the independent variable in **DC**. Previously only phase could be used. Use of time as the independent variable allows solutions for the ephemeris parameters  $t_0$ ,  $P$ , and  $dP/dt$ , as well as the apsidal motion parameter  $d\omega/dt$ . *More extensive use of this provision might well be made, as it nicely complements ephemerides from eclipse timings. Baselines in time are substantially different for whole curve and eclipse timing ephemerides of most EB's, so application of both procedures often enhances perspectives.*
9. The stepped independent variable in **LC** now can be either time or phase. Previously only phase could be stepped. If time is the stepped variable, phase is computed and also listed. If phase is the stepped variable, time is computed and also listed.
10. **DC** now can read the observational input in 1, 2, 3, 4, or 5 data triplets per line, according to the value of NPPL. The old version read only 5 triplets per line.
11. Input and output formats for some parameters of **LC** and **DC** have been expanded to more digits. Some quantities that are likely to range over many orders of magnitude are now entered and printed in D format. See the sample input data sets in Appendices A (p. 48) and B (p. 50) for examples.
12. The argument of periastron,  $\omega$ , now is in radians rather than degrees. The new parameter  $d\omega/dt$  is in radians per day (since  $t$  is Julian Date in days).
13. Spot longitudes, latitudes, and angular radii now are in radians, as are their DEL's and corrections. Previously they were in degrees.

14. The **DC** output format for solution results has been changed and should now be more convenient.
15. **LC** now generates coordinates of plane of sky projected images for use with an external plot program (MPAGE=5).
16. Error estimates printed by **DC** are now standard errors. Previously they were probable errors. Standard errors seem to be more commonly used in the literature than probable errors.
17. Simulated observational scatter can be applied to the light curves computed by **LC**. Similar provisions for radial velocities and line profiles were not then included.
18. The **DC** output now lists the corrected parameter values in addition to the corrections. Previously one had to apply the corrections by hand – the idea being to make certain that users compare new and old values at every iteration. In deference to the significant amount of grumbling generated by that practice, the machine now does the addition. However, **DC** still does only one iteration in a given submission.

## 20. Summary: Differences Between 2003 and 1998 Versions

1. The bandpass-based radiative prescription that is discussed extensively in §12 (pages 30–31) replaces the much simpler previous one based on effective wavelength. The (Kurucz) atmospheres are newer,  $\log g$  is now an atmosphere parameter (allowing for handling giants, sub-giants, etc., in addition to main sequence stars), and 19 chemical compositions can be specified.
2. Another significant change concerns **MODE=3** operation, where  $A_2$ ,  $g_2$ ,  $x_2$ , and  $y_2$  now are free parameters, not set equal to  $A_1$ ,  $g_1$ ,  $x_1$ , and  $y_1$ . Since  $T_2$  may differ considerably from  $T_1$  in mode 3, it seems logical to eliminate those constraints.
3. Input now is via **LC** and **DC** files with standard names that are accessed by **OPEN** statements (FORTRAN). The 1998 and earlier versions utilized the UNIX or LINUX command line syntax, e.g. `lcjob.exe < algolin.d1 > algolout.d1`, but that syntax does not work in some computing environments. The 2003 version of **LC** automatically looks for a file called `lcin.active`, so the idea is to copy `algolin.d1` (or whatever file) into `lcin.active`. The **DC** program looks for a file called `dcin.active`. Of course, persons who like the UNIX command line scheme can just comment out the relevant **LC** and **DC** **OPEN** statements. **LC** and **DC** output goes to files called `lcout.active` and `dcout.active`. Those files are usually copied or re-named to avoid being overwritten in the next **LC** or **DC** run.



## 21. Summary: Differences Between 2007 and 2003 Versions

1. Light-time and velocity shifts due to 3b's are now included, with six corresponding adjustable parameters added to **DC**.
2. **LC** and **DC** now interpolate (locally) in  $[T_{eff}, \log g]$  for  $x$  and  $y$  limb darkening coefficients from the Van Hamme (1993) tables for any of 19 compositions ( $[M/H]$ ) if the law-specifying integer (**LD1**, **LD2**) is negative (§9.1, p. 22). The programs adopt the input coefficients if the integer is positive, so fixed limb darkening is still an option.
3. **DC** now has 60 parameter channels rather than the previous 35 and has 40 adjustable parameters rather than the previous 34. Channels 1 to 29 have the same parameter assignments as before, while the previous channels 30 to 35 are now assigned to 3b parameters. The band-dependent parameters ( $L_1, L_2, x_1, x_2, \ell_3$ ) have been moved from channels 31-35 into channels 56-60. Channels 36 to 55 can be assigned to other parameters, according to needs, via minor changes in the **DC** main program.
4. **DC** now can have either light or magnitude input. Magnitudes are converted internally to light and solutions are always done in light (flux).
5. **LC**'s radial velocity curves now can have Gaussian scatter with a specified standard deviation. Formerly only light curves could have scatter.
6. The "Parameter Wrapper" contributed by A. Prša facilitates array re-dimensioning. Although re-dimensioning has always required changes only in the **LC** and **DC** main programs, the changes needed to be done individually for many quantities. The Parameter Wrapper greatly reduces the number of needed changes and also improves organization by grouping quantities according to array size.
7. **DC** now prints phase along with the un-weighted and weighted observational equations (first column) so as to allow easy graphing of derivatives and residuals vs. phase.
8. The observational equations of partial derivatives and residuals (both unweighted and weighted) are now in D format (formerly they were in F format).
9. Some I/O quantities that previously were in F format are now in D format (to allow for possible very large or small values).
10. **LC** now has an output column labeled "direct magnitude" that is  $-2.5 \log_{10} \ell$ , where  $\ell = \ell_1 + \ell_2 + \ell_3$  from column 5 of **LC** output. No constant is added to the direct magnitudes. The purpose is for **LC** to have the exact analog of **DC**'s magnitude input, mainly for making magnitude graphs that compare observed and computed light curves. **LC**'s preceding column, labeled "set-level magnitude" allows a light curve to be pinned at a specified level at the normalization phase.

## 22. Summary of [0,1] Control Integers

Integer	0	1
IPB	normal value	decouple $L$ from $T$
IFAT1, IFAT2	blackbody	atmospheres
IFSMV1, IFSMV2	spots fixed in longitude	spots move in longitude
IFDER	not print derivative matrix	print derivative matrix
IFM	not print normal equations	print normal equations
IFR	not print radii & their derivatives	print radii and their derivatives
IF3B	ignore third body parameters	utilize third body parameters
ISYM	asymmetrical partial derivatives	symmetrical partial derivatives
ICOR1, ICOR2	not apply RV proximity corrections	apply RV proximity corrections
KDISK	not use disk scratch pad	use disk scratch pad
KEEP	adjust a given parameter	keep parameter fixed
MAGLITE	<b>DC</b> input is flux (light)	<b>DC</b> input is magnitude

## 23. Summary of [0,1,2] & [1,2] Control Integers

Integer	0	1	2
JDPHS	n.a.	indep. variable is time	indep. variable is phase
NOISE	no level-dependent weights	scatter scales with $\sqrt{level}$	scatter scales with $level$
KO	<b>DC</b> does base set only	read from scratch pad	write on scratch pad
MREF	n.a.	approximate reflection	detailed reflection

## 24. Summary of [1,2,3] & [1,2,3,4,5] Control Integers

Integer	1	2	3	4	5
LD1, LD2	linear cosine law	log law	square root law	n.a.	n.a.
MPAGE	light curves	velocity curves	line profiles	radii vs. phase	pictures

## 25. Acknowledgments

Andrej Prša alerted us to several bugs that are now fixed, and also contributed a block of FORTRAN statements that make re-dimensioning easier than before, in addition to suggestions for several other improvements that were implemented. F.S. Bajo also pointed out a bug. Thanks continue to the many persons who helped with suggestions, identification of bugs, and direct testing of the 1998 and earlier versions, as cited in the 1992 and 1998 documentation booklets. Suggestions for improvements in this booklet were received from Josef Kallrath.

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## A. Sample LC Input File

Below is a sample **LC** input file with four input blocks. The first block is for a light curve example with a third body. The second block also is for a light curve and has two dark spots on star 1 and one hot spot on star 2. The third block is for a light curve with two light-attenuating clouds. The fourth block is a MPAGE=3 (line profiles) sample.

```

1 1 1 1 1 1 1 1 +3 +3
1 0034999.509081 0.1485377885d+01 -.736140d-10 +000.0000 .0000d-00 2 138472375.
  047524.500000 0047525.900000 00000.010000 -000.250000 +000.750000 +000.010000 +000.250000
02 0 1 1 060 060 +0006.000000 +.000000d-05 .000000 0100.00
.000000 0.130338d+02 +001.0000 +001.0000 +000.0730 +088.509 01.000 01.000 000.00
02.5000 01.5722 +1.000 +1.000 3.221320d+00 3.514090d+00 0.413919d+00 +0.427 +0.594 +0.367 +0.195
0.256218d+03 0.1191135d+03 +089.00000 0.000000 3.1996553 0034963.68442000
07 010.07625 004.00001 -0.109 -0.070 +0.595 +0.608 00.0981 .0000d-00 +10.000 01.0000 00.550000
300.
300.
150.
1 1 1 1 1 1 1 0 -3 -3
2 0040204.395925 0.3430969559d+01 -.322000d-09 +000.0000 .0000d-10 1 138472375.
0044937.000000 0044939.500000 00000.100000 -000.250000 +000.749000 +000.010000 +000.250000
02 0 1 1 030 030 +0004.082872 +.59200d-05 .000000 0100.00
.16379 0.172439d+02 +001.4108 +001.4108 -000.0681 +088.903 01.000 01.000 000.00
01.1500 00.9888 +1.000 +1.000 0.777681d+01 0.819531d+01 0.766490d-00 +0.566 +0.530 +0.179 +0.173
0.256218d+03 0.1191135d+03 +089.00000 0.000000 3.1996553 0034963.68442000
07 008.35277 003.61127 -0.060 -0.019 0.712 0.724 00.0412 .0000d-00 +10.000 01.0000 00.550000
  0.38994 0.28469 0.34558 0.83023
  0.57994 2.35275 0.19224 0.91461
300.
  1.45151 0.06236 0.22542 1.12987
300.
150.
1 1 1 1 1 1 1 0 +2 +2
1 0024820.000000 0.9565000000d+03 0.000000d-00 +000.0000 .0000d-10 1 138472375.
0045000.000000 0050000.000000 00100.000000 -000.250000 +000.750000 +000.250000 +000.250000
04 0 1 0 030 030 +0001.572872 +.000000d-00 .000000 0001.00
.000000 0.700000d+03 +001.0000 +001.0000 000.0000 +030.000 00.300 01.000 000.00
00.2500 40.0000 +0.100 +1.000 0.400000d+01 0.100000d+05 0.200000d-00 +0.500 +0.600 +0.000 +0.000
0.256218d+03 0.1191135d+03 +089.00000 0.000000 3.1996553 0034963.68442000
06 010.35277 000.61127 0.826 0.535 -0.225 0.281 00.0000 .0000d-00 +12.000 01.0000 00.440000
300.
300.
  000.0000 000.0500 000.0000 0.3500 0.0004d+02 001.0000 1.500d+09 001.3000 -4.000
  000.0100 000.0000 000.0000 0.6500 0.0007d+02 001.0000 2.000d+09 001.3000 -4.000
150.
3 1 1 1 1 1 1 0 +3 +3
2 0040204.396739 0.3430968357d+01 0.000000d-00 +000.0000 .0000d-10 1 138472375.
0050698.100000 0050703.000000 00000.002000 +000.455000 +000.455000 +000.001000 +000.250000
06 0 1 1 030 030 +0004.076719 +.70194d-05 .000000 0001.00
.16346 0.172393d+02 +005.0000 +005.0000 -006.8000 +089.057 01.000 01.000 000.00
01.1500 00.9911 +1.000 +1.000 0.778655d+00 0.818142d+00 0.766310d-00 +0.500 +0.600 +0.000 +0.000
0.256218d+03 0.1191135d+03 +089.00000 0.000000 3.1996553 0034963.68442000
07 008.30199 003.61127 -0.060 -0.014 0.712 0.723 00.0445 .0000d-00 +10.000 01.0000 00.500000
0.10000d-03 000.9900 -0005.00 03
00.447148 1.50000d-04 +00.80000 +001

```

00.446990 1.45000d-04 +00.70000 +002  
 00.448115 1.40000d-04 +00.50000 -001  
 -1.  
 0.10000d-04 001.0000 -0000.00 03  
 00.447100 1.20000d-05 +00.80000 +000  
 -1.  
 300.  
 300.  
 150.  
 9

MPAGE,NREF,MREF,IFSMV1,IFSMV2,ICOR1,ICOR2,IF3B,LD1,LD2  
 8(I1,1X),I2,1X,I2  
 JDPHS,HJD0,PERIOD,DPDT,PSHIFT,STDEV,NOISE,SEED  
 I1,F15.6,D17.10,D13.6,F10.4,D10.4,I2,F11.0  
 HJDST,HJDSP,HJDIN,PHSTRT,PHSTOP,PHIN,PHN  
 F14.6,F15.6,F13.6,4F12.6  
 MODE,IPB,IFAT1,IFAT2,N1,N2,PERR0,DPERDT,THE,VUNIT  
 4I2,2I4,F13.6,D12.5,F7.5,F8.2  
 E,A,F1,F2,VGA,XINCL,GR1,GR2,ABUNIN  
 F6.5,D13.6,2F10.4,F10.4,F9.3,2F7.3,F7.2  
 TAVH,TAVC,ALB1,ALB2,POTH,POTC,RM,XBOL1,XBOL2,YBOL1,YBOL2  
 F7.4,1X,F7.4,2F7.3,3D13.6,4F7.3)  
 A3B,P3B,XINC3B,E3B,PERR3B,TC3B → values have no effect when IF3B=0  
 D12.6,D14.7,F11.5,F9.6,F10.7,F17.8  
 IBAND,HLUM,CLUM,XH,XC,YH,YC,EL3,OPSF,ZERO,FACTOR,WL  
 I3,2F10.5,4F7.3,F8.4,D10.4,F8.3,F8.4,F9.6  
 BINWM1,SC1,SL1,NF1 → star 1 line profile parameters, for MPAGE=3 only  
 D11.5,F9.4,F9.2,I3  
 WLL1,EWID1,DEPTH1,KKS → star 1 line profile parameters, for MPAGE=3 only  
 F9.6,D12.5,F10.5,I5  
 BINWM2,SC2,SL2,NF2 → star 2 line profile parameters, for MPAGE=3 only  
 D11.5,F9.4,F9.2,I3  
 WLL2,EWID2,DEPTH2,KKS → star 2 line profile parameters, for MPAGE=3 only  
 F9.6,D12.5,F10.5,I5  
 XLAT,XLONG,RADSP,TEMSP → spot parameters  
 4F9.5  
 XCL,YCL,ZCL,RCL,OP1,FCL,EDENS,XMUE,ENCL → cloud parameters  
 3F9.4,F7.4,D11.4,F9.4,D11.3,F9.4,F7.3

## B. Sample DC Input File

```

+0.2d-1 +0.2d-1 +0.2d-1 +0.2d-1 +0.2d-1 +0.2d-1 +0.2d-1 +0.2d-1
+1.0d-1 +1.0d-2 +1.0d-1 +1.0d-1 +1.0d-1 +1.0d-4 -1.0d-1 +1.0d-2 +1.0d-2 +1.0d-3 +1.0d-3
+5.0d-2 +5.0d-0 +1.0d-2 +1.0d-2 +1.0d-3 +1.0d-2 +1.0d-2 +1.0d-2 +1.0d-2
1111 1111 0111110 01110 11000 00010 01110 11111 11111 11111 11111 01110 1 1 1 1.000d-05
1 0 2 0
1 1 03 2 0 1 3
1 1 1 1 1 1 1 -3 -3 0
1 34999.509081 0.1485377885E+01 -0.736140E-10 0.0000
2 0 1 1 060 060 030 030 3.141593 0.00000E+00 0.00000 300.000
.00000 0.130338E+02 1.0000 1.0000 0.2190 88.509 1.000 1.000 0.000
2.5000 1.5722 1.000 1.000 0.322132E+01 0.351409E+01 0.413919E+00 0.427 0.594 0.367 0.195
0.256218E+03 0.1191135E+03 +089.00000 0.000000 3.1996553 34963.68442000
7 1.00763d+01 2.04825d+00 -0.109 -0.070 0.595 0.608 0.000E+00 0.23200E-01 0.550000
7 1.00763d+01 2.04825d+00 -0.109 -0.070 0.595 0.608 0.000E+00 0.30800E-01 0.550000
7 10.07625d+00 2.04825d+00 -0.109 -0.070 0.595 0.608 0.0981d+00 0.000E+00 1 0.65780E-02 0.550000 0.07000 0.43000 0.57000 0.93000
6 10.18717d+00 2.04825d+00 -0.116 -0.092 0.670 0.727 0.0902d+00 0.000E+00 1 0.71600E-02 0.440000 0.07000 0.43000 0.57000 0.93000
5 10.90308d+00 2.04825d+00 -0.074 0.003 0.618 0.568 0.0598d+00 0.000E+00 1 0.86580E-02 0.360000 0.07000 0.43000 0.57000 0.93000
300.00000
300.00000
150.
19725.80540 -0.3500 0.141 19727.80270 0.3500 0.187 19729.77151 0.2000 0.164
...
48611.85320 -0.3900 0.623 48699.62060 -0.3300 0.623 0.00000 0.0000 0.000
-10002.00000 0.0000 0.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
42030.67120 -0.3867 0.615 42030.71910 -0.5833 1.093 42036.69060 -0.6533 0.038
...
48312.61300 -0.9500 1.000 48343.63190 -0.8333 1.000 48611.85320 1.2567 1.000
-10003.00000 0.0000 0.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
42041.68570 0.7430 1.000 42038.71510 0.7444 1.000 42041.68940 0.7424 1.000
...
42038.71100 0.7430 1.000 42041.68190 0.7417 1.000 0.00000 0.0000 0.000
-10002.00000 0.0000 0.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
42041.68520 0.7316 1.000 42038.71590 0.7343 1.000 42041.68900 0.7289 1.000
...
42050.58600 0.7329 1.000 42041.67390 0.7316 1.000 42047.61590 0.7363 1.000
-10003.00000 0.0000 0.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
42041.68480 0.7161 1.000 42038.71550 0.7254 1.000 42044.66470 0.7175 1.000
...
42038.71050 0.7228 1.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
-10001.00000 0.0000 0.000 0.00000 0.0000 0.000 0.00000 0.0000 0.000
1111 1111 0111110 01110 11000 00110 01110 11111 11111 11111 11111 01110 0 1 1 1.000d-05
1111 1111 0111110 11110 11000 00010 01110 11111 11111 11111 11111 01111 0 1 0 1.000d-05
2

```

(DEL(I),I=1,8)  
 11(1X,D7.1)  
 (DEL(I),I=9,14),(DEL(I),I=16,20)  
 11(1X,D7.1)  
 (DEL(I),I=21,25),(DEL(I),I=56,59)  
 11(1X,D7.1)  
 (KEP(I),I=1,60),IFDER,IFM,IFR,XLAMDA  
 1X,2(4I1,1X),7I1,1X,9(5I1,1X),I1,1X,I1,1X,I1,D10.3  
 KSPA,NSPA,KSPB,NSPB  
 4I3  
 IFVC1,IFVC2,NLC,KO,KDISK,ISYM,NPPL  
 I1,1X,I1,1X,5I2  
 NREF,MREF,IFSMV1,IFSMV2,ICOR1,ICOR2,IF3B,LD1,LD2,MAGLITE  
 7(I1,1X),2(I2,1X),I1  
 JDPHS,HJD0,PERIOD,DPDT,PSHIFT  
 I1,F15.6,D17.10,D14.6,F10.4  
 MODE,IPB,IFAT1,IFAT2,N1,N2,N1L,N2L,PERR0,DPERDT,THE,VUNIT  
 4I2,4I4,F13.6,D12.5,F8.5,F9.3  
 E,A,F1,F2,VGA,XINCL,GR1,GR2,ABUNIN  
 F6.5,D13.6,2F10.4,F10.4,F9.3,2F7.3,F7.2  
 TAVH,TAVC,ALB1,ALB2,PHSV,PCSV,RM,XBOL1,XBOL2,YBOL1,YBOL2  
 F7.4,F8.4,2F7.3,3D13.6,4F7.3  
 A3B,P3B,XINC3B,E3B,PERR3B,TC3B  
 D12.6,D14.7,F11.5,F9.6,F10.7,F17.8  
 IBAND,HLA,CLA,X1A,X2A,Y1A,Y2A,OPSFA,SIGMA,WLA  
 I3,2D13.5,4F7.3,D10.3,D12.5,F10.6  
 IBAND,HLA,CLA,X1A,X2A,Y1A,Y2A,EL3A,OPSFA,NOISE,SIGMA,WLA,SPHAS1,SPHAS2,SPHAS3,SPHAS4  
 I3,2D13.5,4F7.3,D12.4,D10.3,I2,D12.5,F10.6,4F8.5  
 XLAT,XLONG,RADSP,TEMSP  
 4F9.5  
 XCL,YCL,ZCL,RCL,OP1,FCL,EDENS,XMUE,ENCL  
 3F9.4,F7.4,D11.4,F9.4,D11.3,F9.4,F7.3  
 (PHJD(in),FLUX(in),WT(in),in=ifirst,last)  
 5(F14.5,F9.4,F8.3)  
 (KEP(I),I=1,60),IFDER,IFM,IFR,XLAMDA  
 1X,2(4I1,1X),7I1,1X,9(5I1,1X),I1,1X,I1,1X,I1,D10.3